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ORIGINAL PAPER

Dominated multilinear operators defined on tensor products of Banach spaces

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Abstract

In this paper, we introduce and study new classes of dominated multilinear operators, which we call $(p; p_1, \ldots, p_n; G_1, \ldots, G_n)$-dominated and $(\tilde{p}; p_1, \ldots, p_n; G_1, \ldots, G_n)$-dominated multilinear operators defined on the tensor product of Banach spaces. Some characterizations of this type of operators are given and we prove some important coincidence results. As an application, we characterize $(p; p_1, \ldots, p_n)$-dominated multilinear operators on $\mathcal{C}(K, G)$ and $(p; p_1, \ldots, p_n)$-dominated multilinear operators in the sense of Dinculeanu on $\mathcal{C}(K, G)$, where K is a compact Hausdorff space and G a Banach space. We also treat the connection between an operator T and its associated operators $T^t, \tilde{T}$ and $T^{\#}$ for certain classes.

Keywords Crossnorm · Dominated multilinear operators · (p, G)-summing operators · $(\tilde{p}, G)$-summing operators · Pietsch domination–factorization theorem

Mathematics Subject Classification 46M05 · 46G25 · 47H60 · 47A80 · 47B10

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1 Introduction and background

Let $n \in \mathbb{N}$ and $E, F, G, E_1, \ldots, E_n, F_1, \ldots, E_n, G_1, \ldots, G_n$ be Banach spaces over $\mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$). We will denote by $\mathcal{L}(E_1, \ldots, E_n; F)$ the Banach space of all bounded multilinear operators from $E_1 \times \cdots \times E_n$ into F equipped with the usual sup norm. We also denote by $\mathcal{C}(K, G)$ the space of all continuous functions $f : K \to G$, where K is a compact Hausdorff space. If $G = \mathbb{K}$, we write simply $\mathcal{C}(K)$ instead of $\mathcal{C}(K, \mathbb{K})$. By $\mathcal{B}_E$, we denote the closed unit ball of E and by E^* its topological dual. Let $1 \leq p < \infty$. By $\ell_p^n(E)$ (resp. $\ell_p^{n,w}(E)$), we denote the Banach space of all absolutely (resp. weakly) p-summable sequences $(x_i)_{i=1}^n$ in E endowed with norm

$$\left\|(x_i)_{i=1}^n\right\|_p^p = \sum_{i=1}^n \|x_i\|^p (\text{resp.} \left\|(x_i)_{i=1}^n\right\|_{p,w}^p = \sup_{x^* \in \mathcal{B}_{E^*}} \sum_{i=1}^n |\langle x^*, x_i\rangle|^p).$$

If $p = \infty$, we take $\left\|(x_i)_{i=1}^n\right\|_\infty = \left\|(x_i)_{i=1}^n\right\|_{\infty,w} = \sup_{1 \leq i \leq n} \|x_i\|$.

Let $E \otimes_\varepsilon G$ be the injective tensor product, i.e., the algebraic tensor product $E \otimes G$ endowed with the injective crossnorm

$$\varepsilon(u) = \sup\left\{|\langle u, x^* \otimes z^*\rangle|, \quad \|x^*\|_{E^*} \leq 1, \|z^*\|_{G^*} \leq 1\right\}, \quad u \in E \otimes G,$$

and $E\widehat{\otimes}_\varepsilon G$ its completion.

We say that [4, 19] a norm α on $E \otimes G$ is a crossnorm if it has the following properties:

1. $\alpha(x \otimes z) \leq \|x\|\|z\|$ for every $x \in E$ and $z \in G$.
2. For every $x^* \in E^*$ and $z^* \in G^*$, the linear functional $x^* \otimes z^*$ on $E \otimes F$ is bounded, and $\|x^* \otimes z^*\| \leq \|x^*\|\|z^*\|$.

Consider $1 \leq p, p_1, \ldots, p_n < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$. A multilinear operator T from $E_1 \times \cdots \times E_n$ into F is called $(p; p_1, \ldots, p_n)$-dominated (see [3, 8, 9, 14]) if there is a constant $C > 0$, such that for any finite sequences $(x_i^k)_{1 \leq k \leq l}$ in E_i $(1 \leq i \leq n)$, we have

$$\left\|(T(x_1^k, \ldots, x_n^k))_{k=1}^l\right\|_p \leq C \prod_{i=1}^n \left\|(x_i^k)_{k=1}^l\right\|_{p_i,w}. \tag{1}$$

The class of all $(p; p_1, \ldots, p_n)$-dominated multilinear operators from $E_1 \times \cdots \times E_n$ into F is denoted by $\Pi_{p;p_1,\ldots,p_n}(E_1, \ldots, E_n; F)$, which is a Banach space equipped with the norm $\pi_{p;p_1,\ldots,p_n}(T)$, the smallest constant C, such that the inequality (1) holds. The case $n = 1$ gives the well-known concept of p-summing operators.

Let $K_1, \ldots, K_n$ be compact Hausdorff topological spaces. Every multilinear operator $T \in \mathcal{L}(\mathcal{C}(K_1)\widehat{\otimes}_\varepsilon G_1, \ldots, \mathcal{C}(K_n)\widehat{\otimes}_\varepsilon G_n; F)$ induces an associated operator $T^\# : \mathcal{C}(K_1) \times \cdots \times \mathcal{C}(K_n) \to \mathcal{L}(G_1, \ldots, G_n; F)$ defined by

$$T^\#(x_1, \ldots, x_n)(z_1, \ldots, z_n) = T(x_1 \otimes z_1, \ldots, x_n \otimes z_n).$$

The origin of this notion, for the linear case, is due to Dinculeanu [5] ($T^\#$ is denoted by T' in [5, page 377]). If $T : \mathcal{C}(K)\widehat{\otimes}_\varepsilon G \longrightarrow F$ is a bounded linear operator, then there is a unique bounded linear operator $T^\# : \mathcal{C}(K) \longrightarrow \mathcal{L}(G, F)$, such that $T(f \otimes z) = T^\#(f)(z)$. Swartz proved [20, Theorem 12] that $T : \mathcal{C}(K)\widehat{\otimes}_\varepsilon G \longrightarrow F$ is 1-summing if, and only if, $T^\# : \mathcal{C}(K) \longrightarrow \Pi_1(G, F)$ is 1-summing and this result was extended by Montgomery-Smith and Saab in [12] to any $\mathcal{L}_\infty$-space. Popa [16] gave necessary and sufficient conditions for an operator on the space $\mathcal{C}(K, G)$ to be (r, p)-absolutely summing, and in [17], he generalized this result to multilinear operators, proving that if $T \in \Pi_{p;p_1,\dots,p_n}(\mathcal{C}(K_1)\widehat{\otimes}_\varepsilon G_1, \dots, \mathcal{C}(K_n)\widehat{\otimes}_\varepsilon G_n; F)$ is $(p; p_1, \dots, p_n)$-dominated, then $T^\# : \mathcal{C}(K_1) \times \cdots \times \mathcal{C}(K_n) \rightarrow \Pi_{p;p_1,\dots,p_n}(G_1, \dots, G_n; F)$ is $(p; p_1, \dots, p_n)$-dominated. He also proved a similar result for multiple summing operators in [18, Theorem 1].

Let us recall the notion of (p, G)-summing linear operators introduced in [10, Exposé I, page 1] and studied in [2, 7], which together with the results in [17] are our main motivations for this paper. We try to generalize these notions to multilinear operators, replacing $\mathcal{C}(K)$ by any Banach space E and the ε-crossnorm by any α-crossnorm.

Consider $1 \leq p < \infty$. An operator T from the injective tensor product $E \otimes_\varepsilon G$ into F is said to be (p, G)-summing if there exists a constant $C > 0$, such that, for all finite sequence $(u_i)_{1 \leq i \leq n}$ in $E \otimes G$, we have

$$\left(\sum_{i=1}^{n} \|T(u_i)\|_F^p\right)^{\frac{1}{p}} \leq C \sup_{x^* \in \mathcal{B}_{E^*}} \left(\sum_{i=1}^{n} \|u_i(x^*)\|_G^p\right)^{\frac{1}{p}}, \tag{2}$$

where $u_i(x^*) = \sum_{j=1}^{m_i} x^*(x_{ij}) z_{ij}$, for $u_i = \sum_{j=1}^{m_i} x_{ij} \otimes z_{ij}$, $x_{ij} \in E$ and $z_{ij} \in G$. By [10], this definition is equivalent to say that: there is a probability measure μ on $(\mathcal{B}_{E^*}, \sigma(E^*, E))$, such that $\|T(u)\|^p \leq C^p \int_{\mathcal{B}_{E^*}} \|u(x^*)\|^p \mathrm{d}\mu(x^*)$ for every $u \in E \otimes_\varepsilon G$. This justifies that the inequality (2) remains true for the sequences in $E\widehat{\otimes}_\varepsilon G$. The smallest of such constants defines the (p, G)-norm of T, denoted by $\pi_p^G(T)$ and the space $\Pi_p^G(E\widehat{\otimes}_\varepsilon G, F)$ of all (p, G)-summing operators is a Banach space endowed with such norm.

Let $T : E\widehat{\otimes}_\varepsilon G \rightarrow F$ be a bounded linear operator. We denote by $\tilde{T}$ the associated linear operator from G to $\mathcal{L}(E; F)$ defined by $\tilde{T}(z)(x) = T(x \otimes z)$, $x \in E$ and $z \in G$ (in [2] $\tilde{T}$ is denote by $T^\# = \Phi(T)$). It was proved by Blasco and Signes in [2, Proposition 1.4] that, if $T : E\widehat{\otimes}_\varepsilon G \rightarrow F$ is (p, G)-summing, then $\tilde{T}(z)$ is p-summing for all $z \in G$.

Later in [6], Elezović introduced the following notion in the setting of tensor product. Let α be a crossnorm on $E \otimes G$ and $1 \leq p < \infty$. An operator T from the tensor product $E \otimes_\alpha G$ into F is said to be $(\tilde{p}, G)$-summing if there exists a positive constant C, such that, for any finite sequence $(u_i)_{1 \leq i \leq n}$ in $E \otimes G$, we have

$$\left(\sum_{i=1}^{n}\|T(u_i)\|_F^p\right)^{\frac{1}{p}} \leq C \sup_{x^*\in\mathcal{B}_{E^*},z^*\in\mathcal{B}_{G^*}}\left(\sum_{i=1}^{n}|\langle u_i(x^*),z^*\rangle|^p\right)^{\frac{1}{p}},$$

where $\langle u_i(x^*),z^*\rangle=\sum_{j=1}^{m_i}x^*(x_{ij})z^*(z_{ij})$, for $u_i=\sum_{j=1}^{m_i}x_{ij}\otimes z_{ij}$, $x_{ij}\in E$ and $z_{ij}\in G$. The smallest of such constants defines the $(\tilde{p},G)$-norm of T, denoted by $\pi_{\tilde{p}}^G(T)$ and the space $\Pi_{\tilde{p}}^G(E\widehat{\otimes}_\alpha G,F)$ of all $(\tilde{p},G)$-summing operators is a Banach space endowed with such norm.

Our main focus in this paper is to develop new classes of multilinear operators on tensor products of Banach spaces, similar to those of (p, G)-summing and $(\tilde{p},G)$-summing linear operators.

The paper is organized as follows. In Sect. 2, we introduce the notions of $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated and $(\tilde{p};p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated multilinear operators for $1\leq p,p_1,\ldots,p_n<\infty$ with $\frac{1}{p}=\frac{1}{p_1}+\cdots+\frac{1}{p_n}$.

We prove a natural analog to "Pietsch domination/factorization theorem" for such classes as the linear case introduced and studied by Maurey in [10]. In Sect. 3, we establish the relationship between the three classes of dominated multilinear operators. Finally, in Sect. 4, we treat the relationship between a multilinear operator T and its associated operators T^t, $\tilde{T}$ and $T^\#$ for the classes of summability cited above. As a consequence, some interesting properties are given concerning certain generalizations of the linear case.

2 Characterization of $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated and $(\tilde{p};p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated multilinear operators

Let α_i be a crossnorm on the space $E_i\otimes G_i$, $1\leq i\leq n$. We introduce the following definition for a multilinear operator $T: E_1\otimes_{\alpha_1}G_1\times\cdots\times E_n\otimes_{\alpha_n}G_n\to F$.

Definition 2.1 Consider $1\leq p,p_1,\ldots,p_n<\infty$ with $\frac{1}{p}=\frac{1}{p_1}+\cdots+\frac{1}{p_n}$. The multilinear operator T is said to be

1. $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated if there exists a constant $C>0$, such that

$$\left(\sum_{k=1}^{l}\left\|T\left(u_1^k,\ldots,u_n^k\right)\right\|_F^p\right)^{\frac{1}{p}}\leq C\prod_{i=1}^{n}\sup_{\|x_i^*\|_{E_i^*}\leq 1}\left(\sum_{k=1}^{l}\left\|u_i^k\left(x_i^*\right)\right\|_{G_i}^{p_i}\right)^{\frac{1}{p_i}} \tag{3}$$

for any finite sequences $u_i^k\in E_i\otimes G_i$, $1\leq i\leq n, 1\leq k\leq l$.

2. $(\tilde{p};p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated if there exists a constant $C>0$, such that

$$\left(\sum_{k=1}^{l}\left\|T\left(u_1^k,\ldots,u_n^k\right)\right\|_F^p\right)^{\frac{1}{p}}\leq C\prod_{i=1}^{n}\sup_{\|z_i^*\|_{G_i^*},\|x_i^*\|_{E_i^*}\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle u_i^k\left(x_i^*\right),z_i^*\right\rangle\right|^{p_i}\right)^{\frac{1}{p_i}} \tag{4}$$

for every choice of elements $u_i^k\in E_i\otimes G_i$, $1\leq i\leq n, 1\leq k\leq l$.

We denote by $\pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T)$ and $\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)$, respectively, the infimum of all constants C in (3) and (4) and by $\Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n;F)$ and $\Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n;F)$ the corresponding spaces of such multilinear operators.

Example Consider $1 \le i \le n$. Then, for every $x_i^* \in E_i^*, z_i^* \in G_i^*$ and $y \in F$, the operator $T = x_1^* \otimes z_1^* \otimes \cdots \otimes x_n^* \otimes z_n^* \otimes y$ defined by

$$(x_1^* \otimes z_1^* \otimes \cdots \otimes x_n^* \otimes z_n^* \otimes y)(u_1,\ldots,u_n) = \langle u_1, x_1^* \otimes z_1^* \rangle \cdots \langle u_n, x_n^* \otimes z_n^* \rangle y$$

is $(\tilde{p};p_1,\ldots,p_n,G_1,\ldots,G_n)$-dominated and

$$\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(x_1^* \otimes z_1^* \otimes \cdots \otimes x_n^* \otimes z_n^* \otimes y) = \|y\| \prod_{i=1}^{n} \|x_i^*\| \|z_i^*\|.$$

Suppose that the α_i $(1 \le i \le n)$ are uniform crossnorms. Let X_i, Z_i, E_i, G_i be Banach spaces and let $A_i : X_i \longrightarrow E_i$, $B_i : Z_i \longrightarrow G_i$ be bounded linear operators. Let $T_i = A_i \otimes_{\alpha_i} B_i : X_i \otimes_{\alpha_i} Z_i \longrightarrow E_i \otimes_{\alpha_i} G_i$ be the unique operator defined by $T_i(a \otimes c) = A_i(a) \otimes B_i(c)$, such that $\|T_i\| \le \|A_i\| \|B_i\|$ (see [4, 19]).

Proposition 2.2 *Let* $T \in \Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n;F)$ *and* h *be a bounded linear operator from* F *to* Y. *Then*

$$h \circ T \circ (T_1,\ldots,T_n) \in \Pi^{Z_1,\ldots,Z_n}_{p;p_1,\ldots,p_n}(X_1 \otimes_{\alpha_1} Z_1,\ldots,X_n \otimes_{\alpha_n} Z_n;Y)$$

and

$$\pi^{Z_1,\ldots,Z_n}_{p;p_1,\ldots,p_n}(h \circ T \circ (T_1,\ldots,T_n)) \le \|h\| \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|A_i\| \|B_i\|.$$

***Remark* 2.3** The same holds for $\Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n;F)$.

Now, we characterize the $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated multilinear operators by giving the Pietsch domination theorem. For the proof, we use the full general Pietsch domination theorem presented by Pellegrino et al. in [13].

Let $X_1,\ldots,X_m, Y, E_1,\ldots,E_k$ be (arbitrary) non-void sets and $\mathcal{H}$ be a family of mappings from $X_1 \times \cdots \times X_m$ to Y. Let $K_1,\ldots,K_t$ be compact Hausdorff topological spaces and $G_1,\ldots,G_t$ be Banach spaces. Suppose that the maps

$$\begin{cases} R_j : K_j \times E_1 \times \cdots \times E_k \times G_j \to [0,+\infty), \quad j = 1,\ldots,t \\ S : \mathcal{H} \times E_1 \times \cdots \times E_k \times G_1 \times \cdots \times G_t \to [0,+\infty) \end{cases}$$

satisfy

(1) For each $x^l \in E_l$ and $b \in G_j$, with $(j,l) \in \{1,\ldots,t\} \times \{1,\ldots,k\}$, the mapping $(R_j)_{x^1,\ldots,x^k,b} \colon K_j \to [0,\infty)$, defined by $(R_j)_{x^1,\ldots,x^k,b}(\varphi_j) = R_j(\varphi_j, x^1,\ldots,x^k,b)$, is continuous.

(2) The following inequalities hold:

$$\begin{cases} R_j(\varphi_j, x^1,\ldots,x^k,\eta_j b^j) \le \eta_j R_j(\varphi_j, x^1,\ldots,x^k,b^j) \\ S(f,x^1,\ldots,x^k,\alpha_1 b^1,\ldots,\alpha_t b^t) \ge \alpha_1\cdots\alpha_t S(f,x^1,\ldots,x^k,b^1,\ldots,b^t). \end{cases}$$

For $R_1,\ldots,R_t$ and S as above and $1 \le p, p_1,\ldots,p_t < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_t}$, a mapping $f \in \mathcal{H}$ is said to be $R_1,\ldots,R_t$-S-abstract $(p_1,\ldots,p_t)$-summing if there is a constant $C > 0$, such that

$$\left(\sum_{i=1}^{n} S(f,x_i^1,\ldots,x_i^k,b_i^1,\ldots,b_i^t)^p\right)^{\frac{1}{p}} \le C \prod_{j=1}^{t} \sup_{\varphi_j \in K_j} \left(\sum_{i=1}^{n} R_j(\varphi_j, x_i^1,\ldots,x_i^k,b_i^j)^{p_j}\right)^{\frac{1}{p_j}}$$

for all $x_1^s,\ldots,x_n^s \in E_s, b_1^j,\ldots,b_n^j \in G_j$ and $(s,j) \in \{1,\ldots,k\} \times \{1,\ldots,t\}$.

Now, we can present the abstract domination theorem.

Theorem 2.4 [13, Theorem 4.6] *A map $f \in \mathcal{H}$ is $R_1,\ldots,R_t$-S-abstract $(p_1,\ldots,p_t)$-summing if, and only if, there are a constant $C > 0$ and Borel probability measures μ_j on K_j, such that*

$$S(f,x^1,\ldots,x^k,b^1,\ldots,b^t) \le C \prod_{j=1}^{t} \left(\int_{K_j} R_j(\varphi_j, x^1,\ldots,x^k,b^j)^{p_j} \mathrm{d}\mu_j\right)^{\frac{1}{p_j}}$$

for all $x^l \in E_l$ and $b \in G_j$, with $(j,l) \in \{1,\ldots,t\} \times \{1,\ldots,k\}$.

From the above theorem, we present the domination theorem concerning the class of $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated multilinear operators.

Theorem 2.5 *Let $1 \le p, p_1,\ldots,p_n < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$. A multilinear operator $T : E_1 \otimes_{\alpha_1} G_1 \times \cdots \times E_n \otimes_{\alpha_n} G_n \to F$ is $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated if, and only if, there are a constant $C > 0$ and probability measures μ_i on $(\mathcal{B}_{E_i^*}, \sigma(E_i^*, E_i))$, such that*

$$\|T(u_1,\ldots,u_n)\|_F \le C \prod_{i=1}^{n} \left(\int_{\mathcal{B}_{E_i^*}} \|u_i(x_i^*)\|_{G_i}^{p_i} \mathrm{d}\mu_i(x_i^*)\right)^{1/p_i} \tag{5}$$

for all $u_i \in E_i \otimes G_i$. Moreover, $\pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T)$ is the smallest of the constants verifying the inequality (5).

Proof Choosing the parameters

$$\begin{cases} t = n \\ E_i = \mathbb{K}, i = 1, \ldots, n \\ G_i = E_i \otimes_{\alpha_i} G_i, i = 1, \ldots, n \\ K_i = \mathcal{B}_{E_i^*}, i = 1, \ldots, n \\ \mathcal{H} = \mathcal{L}(E_1 \otimes_{\alpha_i} G_1, \ldots, E_n \otimes_{\alpha_n} G_n; F) \\ S(T, \lambda_1, \ldots, \lambda_n, u_1, \ldots, u_n) = \|T(u_1, \ldots, u_n)\| \\ R_i(x_i^*, \lambda_1, \ldots, \lambda_n, u_i) = \|u_i(x_i^*)\|, i = 1, \ldots, n, \end{cases}$$

we can easily conclude that $T : E_1 \otimes_{\alpha_1} G_1 \times \cdots \times E_n \otimes_{\alpha_n} G_n \to F$ is $(p; p_1, \ldots, p_n; G_1, \ldots, G_n)$-dominated if, and only if, it is $R_1, \ldots, R_n$-S-abstract $(p_1, \ldots, p_n)$-summing. Theorem 2.4 tells us that T is $R_1, \ldots, R_n$-S-abstract $(p_1, \ldots, p_n)$-summing if, and only if, there exist a positive constant C and probability measures μ_i on K_i, $i = 1, \ldots, n$, such that

$$S(T, \lambda_1, \ldots, \lambda_n, u_1, \ldots, u_n) \leq C \prod_{i=1}^{n} \left(\int_{K_i} R_i(x_i^*, \lambda_1, \ldots, \lambda_n, u_i)^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{\frac{1}{p_i}}.$$

Consequently

$$\|T(u_1, \ldots, u_n)\|_F \leq C \prod_{i=1}^{n} \left(\int_{K_i} \|u_i(x_i^*)\|^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{1/p_i},$$

and this concludes the proof. □

Theorem 2.6 (Factorization theorem) *Consider* $1 \leq p, p_1, \ldots, p_n < \infty$ *with* $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$. *Then, the following properties are equivalent:*

1. *A multilinear operator* T *is in* $\Pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \otimes_{\alpha_1} G_1, \ldots, E_n \otimes_{\alpha_n} G_n; F)$.
2. *There exist Banach spaces* $F_1, \ldots, F_n$, (p_i, G_i)*-summing linear operators* $T_i \in \Pi_{p_i}^{G_i}(E_i \otimes_{\alpha_i} G_i; F_i)$ $(1 \leq i \leq n)$ *and an* n*-linear mapping* $S \in \mathcal{L}(F_1, \ldots, F_n; F)$, *such that the following diagram commutes:*

$$\begin{array}{ccccccc} E_1 \otimes_{\alpha_1} G_1 & \times & . \; . \; . & \times & E_n \otimes_{\alpha_n} G_n & \xrightarrow{T} & F. \\ T_1 \downarrow & & & & T_n \downarrow & S \nearrow & \\ F_1 & \times & . \; . \; . & \times & F_n & & . \end{array}$$

Proof First, we prove the converse. Let $u_i \in E_i \otimes G_i$ $(1 \leq i \leq n)$. If T has such a factorization, we have

$$\begin{aligned}\|T(u_1,\ldots,u_n)\| &= \|S(T_1(u_1),\ldots,T_n(u_n))\| \\ &\le \|S\|\prod_{i=1}^{n}\|T_i(u_i)\|.\end{aligned}$$

Since T_i is (p_i, G_i)-summing, then, by Theorem 2.5 for $n = 1$, there is a probability measure μ_i on $(\mathcal{B}_{E_i^*}, \sigma(E_i^*, E_i))$, such that, for all $u_i \in E_i \otimes G_i$

$$\|T_i(u_i)\| \le \pi_{p_i}^{G_i}(T_i)\left(\int_{\mathcal{B}_{E_i^*}} \left\|u_i(x_i^*)\right\|^{p_i} \mathrm{d}\mu_i(x_i^*)\right)^{\frac{1}{p_i}}.$$

Consequently

$$\|T(u_1,\ldots,u_n)\| \le \|S\|\prod_{i=1}^{n}\pi_{p_i}^{G_i}(T_i)\left(\int_{\mathcal{B}_{E_i^*}} \left\|u_i(x_i^*)\right\|^{p_i} \mathrm{d}\mu_i(x_i^*)\right)^{\frac{1}{p_i}}.$$

Therefore, by Theorem 2.5, T is $(p; p_1,\ldots,p_n, G_1,\ldots,G_n)$-dominated and $\pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T) \le \|S\|\prod_{i=1}^{n}\pi_{p_i}^{G_i}(T_i)$.

To prove the first implication, take $T \in \Pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n; F)$. Therefore, there are, by Theorem 2.5, probability measures μ_i on $(\mathcal{B}_{E_i^*}, \sigma(E_i^*, E_i))$, such that, for all $u_i \in E_i \otimes G_i$, we have

$$\|T(u_1,\ldots,u_n)\| \le \pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T)\prod_{i=1}^{n}\left(\int_{\mathcal{B}_{E_i^*}} \left\|u_i(x_i^*)\right\|_{G_i}^{p_i} \mathrm{d}\mu_i(x_i^*)\right)^{1/p_i}.$$

Now, we consider the operator $T_i^0 : E_i \otimes_{\alpha_i} G_i \to L_{p_i}\left(\mathcal{B}_{E_i^*}, \mu_i, G_i\right)$ given by $T_i^0(u_i) = u_i(\cdot)$ (where $L_{p_i}\left(\mathcal{B}_{E_i^*}, \mu_i, G_i\right)$ is the space of the p_i-Bochner integrable functions). Notice that

$$\begin{aligned}\left\|T_i^0(u_i)\right\| &= \left(\int_{\mathcal{B}_{E_i^*}} \left\|u_i(x_i^*)\right\|^{p_i} \mathrm{d}\mu_i(x_i^*)\right)^{\frac{1}{p_i}} \\ &\le \sup_{\|x_i^*\| \le 1} \left\|u_i(x_i^*)\right\| \\ &\le \sup_{\|x_i^*\| \le 1, \|z_i^*\| \le 1} \left|\left\langle u_i(x_i^*), z_i^*\right\rangle\right| \\ &\le \alpha_i(u_i),\end{aligned}$$

for all $u_i \in E_i \otimes G_i$. Let F_i be the closure in $L_{p_i}(\mathcal{B}_{E_i^*}, \mu_i, G_i)$ of the range of T_i^0, i.e., $F_i = \overline{T_i^0(E_i \otimes_{\alpha_i} G_i)}^{L_{p_i}(\mathcal{B}_{E_i^*}, \mu_i, G_i)}$ and $T_i : E_i \otimes_{\alpha_i} G_i \to F_i$ be the extension operator. Note that T_i is (p_i, G_i)-summing with $\pi_{p_i}^{G_i}(T_i) \le 1$. Let S_0 be the multilinear operator defined on $T_1^0(E_1 \otimes_{\alpha_1} G_1) \times \cdots \times T_n^0(E_n \otimes_{\alpha_n} G_n)$ by

$$S_0\big(T_1^0(u_1),\ldots,T_n^0(u_n)\big) = T(u_1,\ldots,u_n).$$

This definition makes sense, because

$$\big\|S_0\big(T_1^0(u_1),\ldots,T_n^0(u_n)\big)\big\| \leq \prod_{i=1}^{n}\big\|T_i^0(u_i)\big\|.$$

It follows that S_0 is continuous on $T_1^0(E_1 \otimes_{\alpha_1} G_1) \times \cdots \times T_n^0(E_n \otimes_{\alpha_n} G_n)$ and has a unique bounded multilinear extension S to $F_1 \times \cdots \times F_n = \overline{T_1^0(E_1 \otimes_{\alpha_1} G_1)}^{L_{p_1}}\Big(\mathcal{B}_{E_1^*},\mu_1,G_1\Big) \times \cdots \times \overline{T_n^0(E_n \otimes_{\alpha_n} G_n)}^{L_{p_n}}\big(\mathcal{B}_{E_n^*},\mu_n,G_n\big)$. Finally, note also that $T = S \circ (T_1,\ldots,T_n)$, where $T_i \in \Pi_{p_i}^{G_i}(E_i \otimes_{\alpha_i} G_i, F_i)(1 \leq i \leq n)$, and this ends the proof. □

Corollary 2.7 *If $T : E_1 \otimes_{\alpha_1} G_1 \times \cdots \times E_n \otimes_{\alpha_n} G_n \longrightarrow F$ is $(p; p_1,\ldots,p_n; G_1,\ldots,G_n)$-dominated, then T admits a multilinear extension $\hat{T}$ from $E_1 \widehat{\otimes}_{\alpha_1} G_1 \times \cdots \times E_n \widehat{\otimes}_{\alpha_n} G_n$ into F which is $(p; p_1,\ldots,p_n; G_1,\ldots,G_n)$-dominated and $\pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T) = \pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(\hat{T})$.*

***Remark* 2.8** By Corollary 2.7, we can replace $E_i \otimes_{\alpha_i} G_i$ in Definition 2.1(1) and Theorem 2.5 with $E_i \widehat{\otimes}_{\alpha_i} G_i$.

Now, we characterize the $(\tilde{p}; p_1,\ldots,p_n; G_1,\ldots,G_n)$-dominated multilinear operators by giving the Pietsch domination theorem. For the proof, we use the full general Pietsch domination theorem as in Theorem 2.5.

Theorem 2.9 *Let $1 \leq p, p_1,\ldots,p_n < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$. A multilinear operator $T : E_1 \otimes_{\alpha_1} G_1 \times \cdots \times E_n \otimes_{\alpha_n} G_n \rightarrow F$ is $(\tilde{p}; p_1,\ldots,p_n; G_1,\ldots,G_n)$-dominated if, and only if, there exist Radon probability measures μ_i on $K_i = \mathcal{B}_{E_i^*} \times \mathcal{B}_{G_i^*}$ and a constant $C > 0$, such that, for all $u_i \in E_i \otimes G_i$, one has*

$$\|T(u_1,\ldots,u_n)\|_F \leq C\prod_{i=1}^{n}\left(\int_{K_i}\big|\langle u_i, x_i^* \otimes z_i^*\rangle\big|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{1/p_i}. \tag{6}$$

Moreover, $\pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T)$ is the smallest of the constants verifying the estimate (6).

Now, we give the factorization theorem for the class of $(\tilde{p}; p_1,\ldots,p_n; G_1,\ldots,G_n)$-dominated multilinear operators. For the proof, we use Theorem 1 in [6] and the same idea in Theorem 2.6. The proof will be omitted.

Theorem 2.10 *Let $1 \leq p, p_1,\ldots,p_n < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$. Then, the following assertions are equivalent:*

1. *An operator $T \in \Pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \otimes_{\alpha_1} G_1,\ldots,E_n \otimes_{\alpha_n} G_n; F)$.*

2. *There exist Banach spaces* $F_1, \ldots, F_n$, $(\tilde{p}_i, G_i)$*-summing linear operators* $T_i \in \Pi_{\tilde{p}_i}^{G_i}(E_i \otimes_{\alpha_i} G_i; F_i)$ $(1 \leq i \leq n)$ *and an n-linear mapping* $S \in \mathcal{L}(F_1, \ldots, F_n; F)$, *such that T admits the factorization* $T = S \circ (T_1, \ldots, T_n)$.

***Remark* 2.11** As in Remark 2.8, we can replace in Definition 2.1(2), Theorems 2.9 and 2.10, $E_i \otimes_{\alpha_i} G_i$ with $E_i \widehat{\otimes}_{\alpha_i} G_i$.

3 Coincidence and inclusion properties

In this section, we investigate the relationships between various classes of dominated multilinear operators and some coincidence results. First, since

$$\sup_{\|z_i^*\|_{G_i^*}, \|x_i^*\|_{E_i^*} \leq 1} \left(\sum_{k=1}^{l} \left| \left\langle u_i^k(x_i^*), z_i^* \right\rangle \right|^{p_i} \right)^{\frac{1}{p_i}} \leq \sup_{\|x_i^*\|_{E_i^*} \leq 1} \left(\sum_{k=1}^{l} \left\| u_i^k(x_i^*) \right\|_{G_i}^{p_i} \right)^{\frac{1}{p_i}}$$

holds for every $u_j^i \in E_i \otimes G_i$, $1 \leq i \leq n$, $1 \leq k \leq l$, we obtain the following proposition.

Proposition 3.1 *If T is* $(\tilde{p}; p_1, \ldots, p_n; G_1, \ldots, G_n)$*-dominated multilinear operator, then T is* $(p; p_1, \ldots, p_n; G_1, \ldots, G_n)$*-dominated and* $\pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T) \leq \pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T)$.

The next theorem is one of our main results. Mimicking the same idea of [2], we have the following result.

Theorem 3.2 *Let* α_i *be an arbitrary crossnorm on* $E_i \otimes G_i$, $1 \leq i \leq n$. *If* $\Pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F) = \Pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$ *then*

$$\mathcal{L}(G_1, \ldots, G_n; F) = \Pi_{p;p_1,\ldots,p_n}(G_1, \ldots, G_n; F).$$

Moreover, $\Pi_{\tilde{p};p_1,\ldots,p_n}^{G,\ldots,G}(E_1 \widehat{\otimes}_{\alpha_1} G, \ldots, E_n \widehat{\otimes}_{\alpha_n} G; G) = \Pi_{p;p_1,\ldots,p_n}^{G,\ldots,G}(E_1 \widehat{\otimes}_{\alpha_1} G, \ldots, E_n \widehat{\otimes}_{\alpha_n} G; G)$ *if, and only if, G is finite-dimensional.*

Proof Suppose that

$$\Pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F) = \Pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F).$$

Let $S \in \mathcal{L}(G_1, \ldots, G_n; F)$. Fix $x_{i,0}^* \in \mathcal{B}_{E_i^*}$ $(1 \leq i \leq n)$ and consider $T : E_1 \widehat{\otimes}_{\alpha_1} G_1 \times \cdots \times E_n \widehat{\otimes}_{\alpha_n} G_n \to F$ given by

$$T(u_1, \ldots, u_n) = S(u_1(x_{1,0}^*), \ldots, u_n(x_{n,0}^*)).$$

So,

$$\left(\sum_{k=1}^{l}\|T(u_1^k,\ldots,u_n^k)\|_F^p\right)^{\frac{1}{p}} = \left(\sum_{k=1}^{l}\left\|S(u_1^k(x_{1,0}^*),\ldots,u_n^k(x_{n,0}^*))\right\|_F^p\right)^{\frac{1}{p}}$$
$$\leq \|S\|\prod_{i=1}^{n}\left(\sum_{k=1}^{l}\left\|u_i^k(x_{i,0}^*)\right\|^{p_i}\right)^{1/p_i}$$
$$\leq \|S\|\prod_{i=1}^{n}\sup_{\|x_i^*\|\leq 1}\left(\sum_{k=1}^{l}\|u_i^k(x_i^*)\|^{p_i}\right)^{1/p_i}$$

and T is $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated multilinear operator. By hypothesis, T is $(\tilde{p};p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated, i.e., there is a positive constant C, such that for all $l\in\mathbb{N}$ and $u_i^k\in E_i\otimes G_i$ $(1\leq k\leq l,\ 1\leq i\leq n)$, we have

$$\left(\sum_{k=1}^{l}\|T(u_1^k,\ldots,u_n^k)\|_F^p\right)^{\frac{1}{p}}\leq C\prod_{i=1}^{n}\sup_{\|z_i^*\|_{G_i^*},\|x_i^*\|_{E_i^*}\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle u_i^k\left(x_i^*\right),z_i^*\right\rangle\right|^{p_i}\right)^{\frac{1}{p_i}}. \tag{7}$$

In particular, if we take $u_i^k=x_{i,0}\otimes z_i^k$ where $x_{i,0}\in\mathcal{B}_{E_i}$, $\langle x_{i,0}^*,x_{i,0}\rangle=1$ $(1\leq i\leq n)$ and use (7), we get

$$\left(\sum_{k=1}^{l}\|S(z_1^k,\ldots,z_n^k)\|_F^p\right)^{\frac{1}{p}}=\left(\sum_{k=1}^{l}\|T(x_{1,0}\otimes z_1^k,\ldots,x_{n,0}\otimes z_n^k)\|_F^p\right)^{\frac{1}{p}}$$
$$\leq C\prod_{i=1}^{n}\sup_{\|z_i^*\|_{G_i^*},\|x_i^*\|_{E_i^*}\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle x_{i,0}\otimes z_i^k,x_i^*\otimes z_i^*\right\rangle\right|^{p_i}\right)^{\frac{1}{p_i}}$$
$$\leq C\prod_{i=1}^{n}\sup_{\|z_i^*\|_{G_i^*},\|x_i^*\|_{E_i^*}\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle x_{i,0},x_i^*\right\rangle\left\langle z_i^k,z_i^*\right\rangle\right|^{p_i}\right)^{\frac{1}{p_i}}$$
$$\leq C\prod_{i=1}^{n}\sup_{\|z_i^*\|_{G_i^*}\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle z_i^k,z_i^*\right\rangle\right|^{p_i}\right)^{\frac{1}{p_i}}.$$

This shows that S is $(p;p_1,\ldots,p_n)$-dominated and completes the proof of the first part.

For the second part, if $\dim(G)$ is finite, the implication is obvious. The second implication is given by the Dvoretsky–Rogers theorem. □

Lemma 3.3 [15, Lemma 1] *For $u^1,\ldots,u^l\in E\otimes_\varepsilon G$, we have*

$$\left\|\left(u^k\right)_{1\leq k\leq l}\right\|_{p,w}=\sup_{\|x^*\|,\|z^*\|\leq 1}\left(\sum_{k=1}^{l}\left|\left\langle u^k,x^*\otimes z^*\right\rangle\right|^p\right)^{\frac{1}{p}}. \tag{8}$$

Proposition 3.4 *Let α_i be an arbitrary crossnorm on $E_i \otimes G_i$, $1 \leq i \leq n$. Then*

1. $\Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F) \subset \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$.
2. $\Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F) = \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F)$.

Proof

1. Let $T \in \Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1 \otimes_{\alpha_1} G_1, \ldots, E_n \otimes_{\alpha_n} G_n; F)$ and take a finite sequences $(u_i^k)_{1 \leq k \leq l}$ in $E_i \otimes G_i$. One has

$$\left(\sum_{k=1}^{l} \left\| T(u_1^k, \ldots, u_n^k) \right\|^p \right)^{\frac{1}{p}} \leq \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \sup_{\|x_i^*\|, \|z_i^*\| \leq 1} \left(\sum_{k=1}^{l} \left| \left\langle u_i^k(x_i^*), z_i^* \right\rangle \right|^{p_i} \right)^{\frac{1}{p_i}}$$

$$= \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \sup_{\|x_i^*\|, \|z_i^*\| \leq 1} \left(\sum_{k=1}^{l} \left| \left\langle u_i^k, x_i^* \otimes z_i^* \right\rangle \right|^{p_i} \right)^{\frac{1}{p_i}}$$

$$\leq \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \sup_{\|u_i^*\|_{(E_i \otimes_{\alpha_i} G_i)^*} \leq 1} \left(\sum_{k=1}^{l} \left| \left\langle u_i^k, u_i^* \right\rangle \right|^{p_i} \right)^{\frac{1}{p_i}}.$$

   Consequently, T is $(p;p_1,\ldots,p_n)$-dominated and $\pi_{p;p_1,\ldots,p_n}(T) \leq \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)$.
2. We can deduce this coincidence from (8).

We conclude by extending T to the entire space by density and the factorization theorem. □

Corollary 3.5 *If an operator $T \in \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F)$, then $T \in \Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F)$.*

The following corollary is an extension of [2, Proposition 1.2].

Corollary 3.6 *If*

$$\Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F) = \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G_1, \ldots, E_n \widehat{\otimes}_{\varepsilon} G_n; F),$$

then

$$\mathcal{L}(G_1, \ldots, G_n; F) = \Pi_{p;p_1,\ldots,p_n}(G_1, \ldots, G_n; F)$$

and $\Pi^{G,\ldots,G}_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G, \ldots, E_n \widehat{\otimes}_{\varepsilon} G; G) = \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\varepsilon} G, \ldots, E_n \widehat{\otimes}_{\varepsilon} G; G)$, if and only if, G is finite-dimensional.

***Remark* 3.7** Since $\mathcal{C}(K_i, G_i) = \mathcal{C}(K_i) \widehat{\otimes}_{\varepsilon} G_i$ $1 \leq i \leq n$ (see [19]), then the space of $(p;p_1,\ldots,p_n)$-dominated in the sense of Dinculeanu (see [17]) actually coincides by (5) with $\Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(\mathcal{C}(K_1) \widehat{\otimes}_{\varepsilon} G_1, \ldots, \mathcal{C}(K_n) \widehat{\otimes}_{\varepsilon} G_n; F)$ and the class of $(p;p_1,\ldots,p_n)$-dominated in the sense of Matos coincides by Proposition 3.4 with $\Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(\mathcal{C}(K_1) \widehat{\otimes}_{\varepsilon} G_1, \ldots, \mathcal{C}(K_n) \widehat{\otimes}_{\varepsilon} G_n; F)$.

The following proposition is an extension of [10, Proposition 1].

Proposition 3.8 *If* $T \in \Pi_{p;p_1,\ldots,p_n}(E_1,\ldots,E_n;F)$, *then* $\overline{T}: E_1\widehat{\otimes}_{\alpha_1}G_1 \times \cdots \times E_n\widehat{\otimes}_{\alpha_n}G_n \longrightarrow F\widehat{\otimes}_\varepsilon G_1\widehat{\otimes}_\varepsilon \cdots \widehat{\otimes}_\varepsilon G_n$ *defined by* $\overline{T}(x_1\otimes z_1,\ldots,x_n\otimes z_n) = T(x_1,\ldots,x_n)\otimes z_1\otimes\cdots\otimes z_n$ *is* $(p;p_1,\ldots,p_n;G_1,\ldots,G_n)$*-dominated.*

Proof Let $u_i = \sum_{j=1}^{m_i} x_{ij}\otimes z_{ij} \in E_i\otimes G_i$. For $z_i^* \in G_i^*$, $1\le i\le n$, we have $u_i(z_i^*) = \sum_{j=1}^{m_i} x_{ij}\langle z_{ij}, z_i^*\rangle$ and

$$\begin{aligned}\overline{T}(u_1,\ldots,u_n) =&\overline{T}\left(\sum_{j=1}^{m_1} x_{1j}\otimes z_{1j},\ldots,\sum_{j=1}^{m_n} x_{nj}\otimes z_{nj}\right)\\ =&\sum_{j=1}^{m_1}\cdots\sum_{j=1}^{m_n}\overline{T}(x_{1j}\otimes z_{1j},\ldots,x_{nj}\otimes z_{nj})\\ =&\sum_{j=1}^{m_1}\cdots\sum_{j=1}^{m_n} T(x_{1j},\ldots,x_{nj})\otimes z_{1j}\otimes\cdots\otimes z_{nj}.\end{aligned}$$

Consider $(z_1^*,\ldots,z_n^*)$ in $(G_1^*\times\cdots\times G_n^*)$. It follows that:

$$\begin{aligned}\overline{T}(u_1,\ldots,u_n)(z_1^*,\ldots,z_n^*) =&\sum_{j=1}^{m_1}\cdots\sum_{j=1}^{m_n} T(x_{1j},\ldots,x_{nj})\langle z_{1j}, z_1^*\rangle\cdots\langle z_{nj}, z_n^*\rangle\\ =&\sum_{j=1}^{m_1}\cdots\sum_{j=1}^{m_n} T(x_{1j}\langle z_{1j}, z_1^*\rangle,\ldots,\langle z_{nj}, z_n^*\rangle x_{nj})\\ =&T(\sum_{j=1}^{m_1} x_{1j}\langle z_{1j}, z_1^*\rangle,\ldots,\sum_{j=1}^{m_n}\langle z_{nj}, z_n^*\rangle x_{nj})\\ =&T(u_1(z_1^*),\ldots,u_n(z_n^*)).\end{aligned}$$

Consider now T in $\Pi_{p;p_1,\ldots,p_n}(E_1,\ldots,E_n;F)$. By [9, Theorem 3.2], there exist a constant $C>0$ and probability measures μ_i on $\mathcal{B}_{E_i^*}$, $1\le i\le n$, such that

$$\begin{aligned}\left\|\overline{T}(u_1,\ldots,u_n)\right\| &= \sup_{\|z_1^*\|,\ldots,\|z_n^*\|\le 1}\left\|\overline{T}(u_1,\ldots,u_n)(z_1^*,\ldots,z_n^*)\right\|\\ &= \sup_{\|z_1^*\|,\ldots,\|z_n^*\|\le 1}\left\|T(u_1(z_1^*),\ldots,u_n(z_n^*))\right\|\\ &\le C\prod_{i=1}^n \sup_{\|z_1^*\|,\ldots,\|z_n^*\|\le 1}\left(\int_{\mathcal{B}_{E_i^*}}\left|\langle u_i(x_i^*), z_i^*\rangle\right|^{p_i} d\mu_i(x_i^*)\right)^{\frac{1}{p_i}}\\ &\le C\prod_{i=1}^n\left(\int_{\mathcal{B}_{E_i^*}}\left\|u_i(x_i^*)\right\|^{p_i} d\mu_i(x_i^*)\right)^{\frac{1}{p_i}}.\end{aligned}$$

This ends the proof using Corollary 2.7. □

The next result generalizes [6, Proposition 4].

Proposition 3.9 *Let $T_i : G_i \to F_i$ be p_i-summing operators and $\overline{T}_i = id_{E_i} \otimes T_i$, $1 \le i \le n$. If $T \in \Pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} F_1, \dots, E_n \widehat{\otimes}_{\alpha_n} F_n; F)$, then $T \circ (\overline{T}_1, \dots, \overline{T}_n) \in \Pi^{G_1,\dots,G_n}_{\tilde{p};p_1,\dots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \dots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$ and $\pi^{G_1,\dots,G_n}_{\tilde{p},p_1,\dots,p_n}(T \circ (\overline{T}_1, \dots, \overline{T}_n)) \le \pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(T) \prod_{i=1}^n \pi_{p_i}(T_i)$.*

Proof For $1 \le i \le n$, $1 \le k \le l$, let $u_i^k = \sum_{j=1}^{m_{ik}} x_{ij}^k \otimes z_{ij}^k \in E_i \otimes G_i$. We have for $x_i^* \in E_i^*$, the equality

$$\overline{T}_i(u_i^k)(x_i^*) = \sum_{j=1}^{m_{ik}} \left\langle x_i^*, x_{ij}^k \right\rangle T_i(z_{ij}^k) = T_i\left(\sum_{j=1}^{m_{ik}} \left\langle x_i^*, x_{ij}^k \right\rangle z_{ij}^k \right) = T_i(u_i^k(x_i^*)).$$

From this equality, we obtain

$$\begin{aligned}
&\left(\sum_{k=1}^{l} \left\| T(\overline{T}_1(u_1^k), \dots, \overline{T}_n(u_n^k)) \right\|^p \right)^{\frac{1}{p}} \\
&\le \pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(T) \prod_{i=1}^{n} \sup_{\|x_i^*\|_{E_i^*} \le 1} \left(\sum_{k=1}^{l} \left\| \overline{T}_i(u_i^k)(x_i^*) \right\|_{F_i}^{p_i} \right)^{\frac{1}{p_i}} \\
&= \pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(T) \prod_{i=1}^{n} \sup_{\|x_i^*\|_{E_i^*} \le 1} \left(\sum_{k=1}^{l} \left\| T_i(u_i^k(x_i^*)) \right\|_{F_i}^{p_i} \right)^{\frac{1}{p_i}} \\
&\le \pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(T) \prod_{i=1}^{n} \pi_{p_i}(T_i) \sup_{\|x_i^*\|_{E_i^*}, \|z_i^*\|_{G_i^*} \le 1} \left(\sum_{k=1}^{l} \left| \left\langle u_i^k(x_i^*), z_i^* \right\rangle \right|^{p_i} \right)^{\frac{1}{p_i}}.
\end{aligned}$$

This shows that $T \circ (\overline{T}_1, \dots, \overline{T}_n)$ is $(\tilde{p}; p_1, \dots, p_n; G_1, \dots, G_n)$-dominated and $\pi^{G_1,\dots,G_n}_{\tilde{p};p_1,\dots,p_n}(T \circ (\overline{T}_1, \dots, \overline{T}_n)) \le \pi^{F_1,\dots,F_n}_{p;p_1,\dots,p_n}(T) \prod_{i=1}^n \pi_{p_i}(T_i)$, which proves the proposition. □

4 Relations with associated operators

For every multilinear operator T from $E_1 \otimes G_1 \times \cdots \times E_n \otimes G_n$ into F, we denote by $T^{\#}$, $\tilde{T}$ and T^t the associated operators from $E_1 \times \cdots \times E_n$ into $\mathcal{L}(G_1, \dots, G_n; F)$, from $G_1 \times \cdots \times G_n$ into $\mathcal{L}(E_1, \dots, E_n; F)$ and from $G_1 \otimes E_1 \times \cdots \times G_n \otimes E_n$ into F, respectively, defined by $T^{\#}(x_1, \dots, x_n)(z_1, \dots, z_n) = T(x_1 \otimes z_1, \dots, x_n \otimes z_n)$, $\tilde{T}(z_1, \dots, z_n)(x_1, \dots, x_n) = T(x_1 \otimes z_1, \dots, x_n \otimes z_n)$ and $T^t(u_1^t, \dots, u_n^t) = T(u_1, \dots, u_n)$, where $u_i = \sum_{j=1}^{m_i} x_{ij} \otimes z_{ij} \in E_i \otimes G_i$, and $u_i^t = \sum_{j=1}^{m_i} z_{ij} \otimes x_{ij} \in G_i \otimes E_i$. Clearly, $T^{\#}$, $\tilde{T}$ and T^t are multilinear operators. In this section, we try to find some relationships between the operator T and their associated operators $T^{\#}$, $\tilde{T}$ and T^t relative to the different variants of summabilities.

We recall (see [1] and [11, Remark 2.2]) that $T \in \mathcal{D}_p^n(E_1, \dots, E_n; F)$, the class of all Cohen strongly p-summing n-linear operators from $E_1 \times \cdots \times E_n$ into F, if there is a positive constant C, such that for all $m \in \mathbb{N}$, $x_i^1, \ldots, x_i^m \in E_i$ and $y_1^*, \ldots, y_m^* \in F^*$, we have

$$\|(\langle T(x_1^j, \ldots, x_n^j), y_j^* \rangle)\|_1 \le C \prod_{i=1}^{n} \left(\sum_{j=1}^{m} \|x_i^j\|_{E_i}^{p_i} \right)^{\frac{1}{p_i}} \sup_{\|y\|_F \le 1} \|(y_j^*(y))\|_{p^*}. \tag{9}$$

We equip $\mathcal{D}_p^n(E_1, \ldots, E_n; F)$ with the norm $d_p^n(\cdot)$, which is the smallest of the constants C verifying (9) and with which it becomes a Banach space.

The Chevet–Saphar crossnorms are defined, for $1 \le p \le \infty$, as follows (see [4, 19]):

$$d_p(u) = \inf\left\{ \left\|(x_i)_{i=1}^n\right\|_{p^*,w} \left\|(y_i)_{i=1}^n\right\|_p : u = \sum_{i=1}^{n} x_i \otimes y_i \in E \otimes F \right\}$$

and

$$g_p(u) = \inf\left\{ \left\|(x_i)_{i=1}^n\right\|_p \left\|(y_i)_{i=1}^n\right\|_{p^*,w} : u = \sum_{i=1}^{n} x_i \otimes y_i \in E \otimes F \right\}.$$

We have $g_p = d_p^t$, for every p, in the sense that $g_p(u) = d_p(u^t)$.

Proposition 4.1 *Let α_i be an arbitrary crossnorm on $E_i \otimes G_i$, $1 \le i \le n$. Then*

1. *A multilinear operator $T \in \Pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$ if, and only if, $T^t \in \Pi_{\tilde{p};p_1,\ldots,p_n}^{E_1,\ldots,E_n}(G_1 \widehat{\otimes}_{\alpha_1} E_1, \ldots, G_n \widehat{\otimes}_{\alpha_n} E_n; F)$.*
2. *A multilinear operator $T \in \mathcal{D}_p^n(E_1 \widehat{\otimes}_{d_{p_1}} G_1, \ldots, E_n \widehat{\otimes}_{d_{p_n}} G_n; F)$ if, and only if, $T^t \in \mathcal{D}_p^n(E_1 \widehat{\otimes}_{g_{p_1}} G_1, \ldots, E_n \widehat{\otimes}_{g_{p_n}} G_n; F)$.*

Proof

1. We can deduce this from the equality $\langle u_i^k(x_i^*), z_i^* \rangle = \langle (u_i^k)^t(x_i^*), z_i^* \rangle$ and we deduce $\pi_{\tilde{p};p_1,\ldots,p_n}^{E_1,\ldots,E_n}(T^t) = \pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(T)$.
2. Follows immediately from the definitions of the Chevet–Saphar norms.

□

***Remark* 4.2** Using Proposition 3.4 , we have $T \in \Pi_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_\varepsilon G_1, \ldots, E_n \widehat{\otimes}_\varepsilon G_n; F)$ if, and only if, $T^t \in \Pi_{p;p_1,\ldots,p_n}(G_1 \widehat{\otimes}_\varepsilon E_1, \ldots, G_n \widehat{\otimes}_\varepsilon E_n; F)$.

***Remark* 4.3** If $T \in \Pi_{p;p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$ in general $T^t \notin \Pi_{p;p_1,\ldots,p_n}^{E_1,\ldots,E_n}(G_1 \widehat{\otimes}_{\alpha_1} E_1, \ldots, G_n \widehat{\otimes}_{\alpha_n} E_n; F)$. Indeed, consider $T \in \mathcal{L}(G, \ldots, G; G) = \Pi_{p;p_1,\ldots,p_n}^{G,\ldots,G}(\mathbb{R} \widehat{\otimes}_{\alpha_1} G, \ldots, \mathbb{R} \widehat{\otimes}_{\alpha_n} G; G)$ but $T^t \notin \Pi_{p;p_1,\ldots,p_n}^{\mathbb{R},\ldots,\mathbb{R}}(G \widehat{\otimes}_{\alpha_1} \mathbb{R}, \ldots, G \widehat{\otimes}_{\alpha_n} \mathbb{R}; G) = \Pi_{p;p_1,\ldots,p_n}(G, \ldots, G; G)$, if $\dim(G)$ is infinite.

Proposition 4.4 *If $T \in \Pi_{\tilde{p};p_1,\ldots,p_n}^{G_1,\ldots,G_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1, \ldots, E_n \widehat{\otimes}_{\alpha_n} G_n; F)$, then*

1. *$T^{\#}(x_1,\ldots,x_n)$ is $(p;p_1,\ldots,p_n)$-dominated for all $(x_1,\ldots,x_n)\in E_1\times\cdots\times E_n$. Moreover, there exist Radon probability measures μ_i on $K_i=\mathcal{B}_{E_i^*}\times\mathcal{B}_{G_i^*}$, such that for all $u_i\in E_i\otimes G_i$, $1\le i\le n$, the following inequality holds*:

$$\pi_{p;p_1,\ldots,p_n}(T^{\#}(x_1,\ldots,x_n))\le C\prod_{i=1}^n\left(\int_{K_i}|\langle x_i,x_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{1/p_i}.$$

2. *$T^{\#}:E_1\times\cdots\times E_n\to\Pi_{p;p_1,\ldots,p_n}(G_1,\ldots,G_n;F)$ is $(p;p_1,\ldots,p_n)$-dominated.*

Proof Since T is $(\tilde{p};p_1,\ldots,p_n;G_1,\ldots,G_n)$-dominated, by Theorem 2.9, there exist Radon probability measures μ_i on $K_i=\mathcal{B}_{E_i^*}\times\mathcal{B}_{G_i^*}$, such that for all $u_i\in E_i\otimes G_i$, $1\le i\le n$, the following inequality holds:

$$\|T(u_1,\ldots,u_n)\|_F\le\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^n\left(\int_{K_i}|\langle u_i(x_i^*),z_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{1/p_i}.$$

1. We would like to show that $T^{\#}(x_1,\ldots,x_n)$ is $(p;p_1,\ldots,p_n)$-dominated for all $x_i\in E_i$, $1\le i\le n$. Let $u_i^k=x_i\otimes z_i^k\in E_i\otimes_{\alpha_i}G_i$ for $1\le k\le l$. Using Hölder's inequality, we obtain

$$\begin{aligned}&\left(\sum_{k=1}^l\left\|T^{\#}(x_1,\ldots,x_n)(z_1^k,\ldots,z_n^k)\right\|_F^p\right)^{\frac{1}{p}}\\
&=\left(\sum_{k=1}^l\left\|T(x_1\otimes z_1^k,\ldots,x_n\otimes z_n^k)\right\|_F^p\right)^{\frac{1}{p}}\\
&\le\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\left(\sum_{k=1}^l\left[\prod_{i=1}^n\left(\int_{K_i}|\langle z_i^k,z_i^*\rangle\langle x_i,x_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{\frac{1}{p_i}}\right]^p\right)^{\frac{1}{p}}\\
&\le\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^n\left(\sum_{k=1}^l\int_{K_i}|\langle z_i^k,z_i^*\rangle\langle x_i,x_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{\frac{1}{p_i}}\\
&\le\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^n\left(\int_{K_i}\left(\sum_{k=1}^l|\langle z_i^k,z_i^*\rangle|^{p_i}\right)|\langle x_i,x_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{\frac{1}{p_i}}\\
&\le\pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\left\|(z_i^k)_{1\le k\le l}\right\|_{p_i,w}\prod_{i=1}^n\left(\int_{K_i}|\langle x_i,x_i^*\rangle|^{p_i}\mathrm{d}\mu_i(x_i^*,z_i^*)\right)^{\frac{1}{p_i}}.\end{aligned}$$

This yields that $T^{\#}(x_1,\ldots,x_n)$ is $(p;p_1,\ldots,p_n)$-dominated for all $x_i \in E_i$, $1 \le i \le n$, and

$$\pi_{p;p_1,\ldots,p_n}(T^{\#}(x_1,\ldots,x_n)) \le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^{n}\left(\int_{K_i}\left|\left\langle x_i,x_i^*\right\rangle\right|^{p_i}\mathrm{d}\mu_i\left(x_i^*,z_i^*\right)\right)^{1/p_i}.$$

2. For the second property, we use the last inequality and Hölder's inequality again. We deduce, for all $(x_1^k,\ldots,x_n^k) \in E_1 \times \cdots \times E_n$, $1 \le k \le l$, that

$$\begin{aligned}
&\left(\sum_{k=1}^{l}\left[\pi_{p;p_1,\ldots,p_n}(T^{\#}(x_1^k,\ldots,x_n^k))\right]^p\right)^{\frac{1}{p}}\\
&\le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\left(\sum_{k=1}^{l}\left[\prod_{i=1}^{n}\left(\int_{K_i}\left|\left\langle x_i^k,x_i^*\right\rangle\right|^{p_i}\mathrm{d}\mu_i\left(x_i^*,z_i^*\right)\right)^{1/p_i}\right]^p\right)^{\frac{1}{p}}\\
&\le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^{n}\left(\sum_{k=1}^{l}\int_{K_i}\left|\left\langle x_i^k,x_i^*\right\rangle\right|^{p_i}\mathrm{d}\mu_i\left(x_i^*,z_i^*\right)\right)^{\frac{1}{p_i}}\\
&\le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^{n}\left(\int_{K_i}\sum_{k=1}^{l}\left|\left\langle x_i^k,x_i^*\right\rangle\right|^{p_i}\mathrm{d}\mu_i\left(x_i^*,z_i^*\right)\right)^{\frac{1}{p_i}}\\
&\le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)\prod_{i=1}^{n}\left\|\left(x_i^k\right)_{1\le k\le l}\right\|_{p_i,w}.
\end{aligned}$$

Thus, $T^{\#} : E_1 \times \cdots \times E_n \to \Pi_{p;p_1,\ldots,p_n}(G_1,\ldots,G_n;F)$ is $(p;p_1,\ldots,p_n)$-dominated and $\pi_{p;p_1,\ldots,p_n}(T^{\#}) \le \pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(T)$.

□

The following proposition can be proved with the same arguments as the previous result.

Proposition 4.5 *If* $T \in \Pi^{G_1,\ldots,G_n}_{\tilde{p};p_1,\ldots,p_n}(E_1\widehat{\otimes}_{\alpha_1}G_1,\ldots,E_n\widehat{\otimes}_{\alpha_n}G_n;F)$, *then*

1. $\tilde{T}(z_1,\ldots,z_n)$ *is* $(p;p_1,\ldots,p_n)$*-dominated for all* $(z_1,\ldots,z_n) \in G_1 \times \cdots \times G_n$.
2. $\tilde{T} : G_1 \times \cdots \times G_n \to \Pi_{p;p_1,\ldots,p_n}(E_1,\ldots,E_n;F)$ *is* $(p;p_1,\ldots,p_n)$*-dominated.*

Using Propositions 3.4, 4.4, and 4.5, we have the next

Corollary 4.6 *If* $T \in \Pi_{p;p_1,\ldots,p_n}(E_1\widehat{\otimes}_{\varepsilon}G_1,\ldots,E_n\widehat{\otimes}_{\varepsilon}G_n;F)$, *then*

1. $\tilde{T}$ *is* $(p;p_1,\ldots,p_n)$*-dominated and* $\tilde{T}(z_1,\ldots,z_n)$ *is* $(p;p_1,\ldots,p_n)$*-dominated.*
2. $T^{\#}$ *is* $(p;p_1,\ldots,p_n)$*-dominated and* $T^{\#}(x_1,\ldots,x_n)$ *is* $(p;p_1,\ldots,p_n)$*-dominated.*

We end this paper by the following proposition.

Proposition 4.7 *If* $T \in \Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1,\ldots,E_n \widehat{\otimes}_{\alpha_n} G_n;F)$, *then* $\tilde{T}(z^1,\ldots,z^n)$ *is* $(p;p_1,\ldots,p_n)$*-dominated.*

Proof Let T be in $\Pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(E_1 \widehat{\otimes}_{\alpha_1} G_1,\ldots,E_n \widehat{\otimes}_{\alpha_n} G_n;F)$, then by Theorem 2.5, there exist probability measures μ_i on $K_i = \mathcal{B}_{E_i^*}$, such that, for every $u_i \in E_i \otimes G_i$, we have

$$\|T(u_1,\ldots,u_n)\|_F \leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \left(\int_{K_i} \|u_i(x_i^*)\|^{p_i}_{G_i} \mathrm{d}\mu_i(x_i^*) \right)^{1/p_i}.$$

For $u_i = x_i \otimes z_i \in E_i \otimes G_i$, by the previous inequality, we obtain

$$\begin{aligned}\|\tilde{T}(z_1,\ldots,z_n)(x_1,\ldots,x_n)\| &= \|T(x_1 \otimes z_1,\ldots,x_n \otimes z_n)\| \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \left(\int_{K_i} \|x_i \otimes z_i(x_i^*)\|^{p_i}_{G_i} \mathrm{d}\mu_i(x_i^*) \right)^{1/p_i} \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\| \prod_{i=1}^{n} \left(\int_{K_i} |x_i^*(x_i)|^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{1/p_i}.\end{aligned}$$

Using Hölder's inequality, we have

$$\begin{aligned}&\left(\sum_{k=1}^{l} \|\tilde{T}(z_1,\ldots,z_n)(x_1^k,\ldots,x_n^k)\|^p \right)^{\frac{1}{p}} \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\| \left(\sum_{k=1}^{l} \left[\prod_{i=1}^{n} \left(\int_{K_i} |\langle x_i^k, x_i^* \rangle|^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{1/p_i} \right]^p \right)^{\frac{1}{p}} \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\| \prod_{i=1}^{n} \left(\sum_{k=1}^{l} \int_{K_i} |\langle x_i^k, x_i^* \rangle|^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{\frac{1}{p_i}} \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\| \prod_{i=1}^{n} \left(\int_{K_i} \sum_{k=1}^{l} |\langle x_i^k, x_i^* \rangle|^{p_i} \mathrm{d}\mu_i(x_i^*) \right)^{\frac{1}{p_i}} \\ &\leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\| \prod_{i=1}^{n} \left\| (x_i^k)_{1 \leq k \leq l} \right\|_{p_i,w},\end{aligned}$$

and this shows $\tilde{T}(z_1,\ldots,z_n)$ is $(p;p_1,\ldots,p_n)$-dominated and $\pi_{p;p_1,\ldots,p_n}(\tilde{T}(z_1,\ldots,z_n)) \leq \pi^{G_1,\ldots,G_n}_{p;p_1,\ldots,p_n}(T) \prod_{i=1}^{n} \|z_i\|$. □

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