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Numerical method for Volterra integro-differential equation by Taylor Collocation

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Dedication

I decicate this work :
To my source of power and strength woh bilived me
my parents **Mostefa** and **FELLOUSSIA Fatima**
To my sunshine my older sister **Samra** ,
my 2 supporting brothers **mahmoud** and **Seif**
and the reson of my smile **Loudjaine** and my nephew **Wassim**
To who shared with me this journey with its lights and shadows **Chaima**
and my lovely friend **Imene**

Sameh Racha

Notations

$\mathbb{R}$: a set of reel numbers .

$\mathbb{C}$: a set of complex numbers .

$\mathbb{N}$: a set of natural numbers .

IDE: Integro-Differential equation .

$VIDE$: Volterra Integro-Differential equation .

VIE: Volterra Integral equation .

$\mathbb{R}_n[X]$: a set of polynomial .

ϵ : $epsilon$

Contents

General Introduction

The Volterra integro-differential equation of the second kind is an equation in which both differential and integral operators appeared in the same equation,its characteristic is that one side of the integration domain is unknown i.e

$$\begin{cases} y^{(n)}(x) = f(x) + \int_0^x k(x,t)y(t)dt, \quad 0 < x < 1 \\ y^{(n-k)}(0) = \alpha. \ k = 1, ..., n. \end{cases}$$

with $k(x,t)$ is a continuous function , $y(x)$ is the required function
and $y^{(n-k)}(0)$ is the initial conditions .
It has an important role in various fields of science such that mathematics , physics ,chemistry and biology .Usually, this type of equation can't be solved analytically so we used the branch of approximation numerical.
A Taylor collocation method is presented the numerically solving the VIDE in terms of Taylor polynomial with using the Taylor collocation points.

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x_i - c)^n$$

where the Taylor coefficients $y^{(n)}(c)$ are the unknown .
The objective of this thesis is to apply the method of Taylor collocation on the Integro-differential equation of Volterra type to find the approaching solution and compare it with the exact solution to verify the correctness of the method .
This thesis is organized in the following way :

the first chapter is a reminder of preliminary of some notions of bases of functional analysis , numerical and the operators such as a Hilbert spaces and collocation and a linear operators .
the second chapter is introductory concepts of integral , differential and Integro-Differential equations , explaining the relationship between them .
the third chapter is concerned with Taylor collocation method for resolution of VIDE. we showed the details of the method for obtaining the equation to form system algebraic and we applied it to an example .
the forth chapter we give some examples to prove the results and compare it with exact solution by the MATLAB programing .

CHAPTER 1

REMAINDERS AND FUNDAMENTAL NOTIONS

In this chapter, we recall certain fundamental preliminary notions and the tools necessary in the memory concerning the functional,the numerical definitions and the operators.

1.1 Functional Analysis Notions

1.1.1 Normed space

Definition 1.1.1. Let E be a vectorial space in the field $\mathbb{K} = \mathbb{R}\ or\ \mathbb{C}$ we say that E is normed vectorial space if it endowed a norm $\|.\|$.

Definition 1.1.2. (**Norm**) We call a norm in the space E any function denoted $\|.\|$ defined from E to values in $\mathbb{R}$ .such as

1. $\|x\| = 0 \Leftrightarrow x = 0$
2. $\|\lambda x\| = |\lambda|\,\|x\| \quad for\ all\ x\ \in\ E, and\ \lambda \in \mathbb{K}$
3. $\|x + y\| \leq \|x\| + \|y\| \quad for\ all\ x, y \in E$

Theorem 1.1.1. All normed space is metrizable

proof
For all x,y $\in E$, we defined the function f by $f(x, y) = \|x - y\|$ we observe that the function is well metric in E because

$$f(x, y) = \|x - y\| = 0 \Rightarrow\ x - y = 0$$

Hence

$$x = y$$

it's obvious to see that f(x,y) is symmetrical

$$f(x, y) = \|x - y\|$$

$$= \|y - x\| = f(y, x)$$

For the triangle inequality , we write

$$f(x, y) = \|x - y\| = \|(x - z) + (z - y)\|$$
$$\leq \|x - z\| + \|z - y\|$$
$$= f(x, z) + f(z, y)$$

1.1.2 Banach space

Definition 1.1.3. We call Banach space $(E, \|.\|)$ all normed vector and completed for the distance deducted from its norm .

Definition 1.1.4. (**Completed space**) Normed space $(E, \|.\|)$ is said completed if all Cauchy sequence x_n with element of E is a convergent sequence in E

$$\forall \epsilon > 0, \exists N_\epsilon \in N, \forall p, q \geq N_\epsilon \Rightarrow \|x_p - x_q\| < \epsilon$$

such as

$$\lim_{n \to +\infty} x_n = x$$

Definition 1.1.5. (**Cauchy sequence**) Let x_n be a sequence of element of normed space $(E, \|.\|)$, we say that the sequence x_n is Cauchy if the following relation is verified

$$\forall \epsilon > 0, \exists N_\epsilon, \forall p, q \geq N_\epsilon \; we \; have \; \|x_p - x_q\| < 0$$

Theorem 1.1.2. Let x_n be a cauchy sequence in normed space $(E, \|.\|)$ contains a subsequence x_{n_k} convergent toward x. Hence the sequence x_n is also convergent toward the same element x

proof

Let x_n be a Cauchy sequence then it comes

$$\forall \epsilon > 0, \exists N_\epsilon, \forall p, q \geq N_\epsilon \; we \; have \; \|x_p - x_q\| < \epsilon$$

we particular for $n_k \geq N_\epsilon$, having

$$\forall p, n_k \geq N_\epsilon \;\; we \; have \; \|x_p - x_{n_k}\| < \epsilon$$

with the convergent of the sequence x_n toward x

$$n_k \geq N_\epsilon, \|x_{n_k} - x\| < \epsilon.$$

where the convergence of the sequence x_n toward x

$$\begin{aligned} \forall p, n_k \geq N_\epsilon, \|x_p - x\| &= \|x_p - x + x_{n_k} - x_{n_k}\| \\ &\leq \|x_p - x_{n_k}\| + \|x_{n_k} - x\| < \epsilon \end{aligned}$$

1.1.3 Hilbert space

Definition 1.1.6. A Hilbert space is a Banach space which contains an inner product .

Definition 1.1.7. (**Inner product**) Let E be a vectorial space in $\mathbb{R}$, an inner product in E is a mapping of $E \times E \in \mathbb{R}$, noted $<.,.>$ verifying the following properties :
$\forall x, y, z \in E$, $\lambda \in \mathbb{R}$

1. $< x, x > \geq 0$
2. $< x, x >= 0 \ \Rightarrow \ x = 0$
3. $< \lambda x, y >= \lambda < x, y >$
4. $< x, y + z >=< x, y > + < x + z >$
5. $< x, y >= \overline{< x, y >}$

Remark 1.1. 1. A vectorial space that endowed an inner product is called an Euclidien space or Prehilbertien space.

2. The definition of inner product give us the following statements .
   - $< x + y, z >=< x, z > + < y, z >$.
   - $< x, \lambda y >= \overline{\lambda} < x, y >$.
3. We can introduce the inner product as a norm defined by :

$$\|x\| = \sqrt{< x, x >}$$

1.2 Numerical Notions

1.2.1 Collocation Method

In mathematics, the collocation method is a method of numerical resolution of integral,differential and integro-differential equations . The idea is choosing a finite dimensional space of candidate solution (in the generally polynomials up to a certain degree) and a certain number of points in the domain (which called the collocation points) and to select the solution which satisfied the equation giving to collocation points .

1.2.2 Taylor series

Taylor's formula permit the approximation of a function several times differentiable in the vicinity of a point by a polynomial whose coefficients depend only on the differentiable of the function at this point .
Standard format :

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n \tag{1.1}$$

where
$y^{(n)}(c)$ are the Taylor coefficients to be determined
(c) is appropriate constant and N is chosen.
The Taylor Collocation point : defined by

$$x_i = a + i\frac{b-a}{N} \quad ; i = 0, 1...., N; \quad x_0 = a, x_1 = b \tag{1.2}$$

Theorem 1.2.1. for all $n \in \mathbb{N}$, the Taylor formela

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n$$

and let the taylor polynomial be $X = (x-c)^n$ which form a base of $\mathbb{R}_n[X]$

Proof Let the Taylor polynomial be

$$X = (x-c)^n; \quad n = 0, 1.....N \tag{1.3}$$

So we will demonstrate that (1.3) form a free familly of $\mathbb{R}_n[X]$
We consider $\lambda_n \in \mathbb{R}; \;\; n = 0, 1...N$ such that :

$$\lambda_n X = 0$$

i.e

$$\lambda_0(x-c)^0 + \lambda_1(x-c)^1 + \ldots + \lambda_N(x-c)^N = 0$$

For $x \neq 0$ we obtain

$$\lambda_0 = \lambda_1 = \ldots = \lambda_N = 0$$

Therefore , $y(x) = (x-c)^n$ is a free familly
we can say that the Taylor's polynomials are linearly independent .

1.3 Operators Notions

1.3.1 Bounded linear operators

Definition 1.3.1. (**The linearity**) Let E and F be two normed spaces , an operator A defined in E to F is called linear if and only if

- Additive condition:

$$\forall \phi_1, \phi_2 \in E \text{ , } we\ have\ A(\phi_1 + \phi_2) = A(\phi_1) + A(\phi_2)$$

- Homogeneous condition :

$$\forall \phi \in E \text{ , } \lambda \in K\ we\ have\ A(\lambda\phi) = \lambda A(\phi)$$

Definition 1.3.2. (**Continuity of linear operator**) Let E and F be two normed spaces , a linear operator A defined in $G \subset$ E to F is called continuous in x_0 of G if :
for all sequence x_n of G converges toward x_0, the sequence $A(x_n)$ converges toward $A(x_0)$ that means

$$\lim_{n\to\infty} A(x_n) = A(\lim_{n\to\infty} x_n) = A(x_0)$$

Remark 1.2. Linear operator A is called continuous in G if it is continuous in each point of the set G

Theorem 1.3.1. Let E,F be two normed spaces , linear operator A defined in a subset $G \subset E$ to F is called continuous everywhere in G if it is continuous in point x_0 of G

Definition 1.3.3. (**Bounded operator**) A linear operator A defined in E to F is called bounded if there exists a positive constant $C > 0$ such that :

$$\|A(x)\|_F \leq C\,\|x\|_E \quad \forall x \in E$$

Theorem 1.3.2. A linear operator A is continuous iff it is bounded

proof

We suppose that the operator A is bounded , we have

$$\|A(x)\|_F \leq C\,\|x\|_E$$

Or

$$\|A(x) - A(0)\|_F \leq C\,\|x - 0\|_E$$

where the continuity of the operator A in 0 such that

$$\lim_{n\to\infty} A(x) = A(0) \;\; where \;\; \lim_{n\to\infty} x = 0$$

which leads to the continuity everywhere .

1.3.2 Integro-Differential operator

Integral operator

Definition 1.3.4. we call integral operator all linear operator A defined in a normed space E to normed space F given by the formula :

$$A\varphi(x) = \int_{G_2} K(x,y)\varphi(y)dy \qquad x \in G_1$$

where
K(x,y) :measurable function defined in measuring set $G_1 \times G_2$ who is called the kernel of integral operator A
$\varphi(y)$: is measurable function defined in G_2

Differential operator

Definition 1.3.5. A linear differential operator in order m is defined by

$$D = \sum_{\alpha=0}^{m} a_\alpha(x) D^\alpha$$

where the $a_\alpha(x)$ are functions of n variable where they are called coefficients of operator D

Remark 1.3. Differential operator is an acting operator in the differentiable functions

- When the function has single variable ;the differential operator is constructed from ordinary derivatives.
- When the function has multiple variable ;the differential operator is constructed form partial derivatives .

when both differential and integral operators appeared in the same equation , we call the operator Integro-Differential operator .

CHAPTER 2

INTRODUCTION TO VOLTERRA INTEGRO-DIFFERENTIEL EQUATION

In this chapter, we remind the integral,differential and integro-differential equations and their classification ,as well as the relationship between them,and we give the Taylor series method for the analytical solution .

2.1 Different types of equations

2.1.1 Integral equation

Definition 2.1.1. we call integral equation all functional equation where the unknown function appears under the sign of integration .

the general form of integral equation with unknown y(x) is giving by :

$$y(x) + \int_E k(x,t)y(t)dy = f(x)$$

where f(x), k(x,t) are giving ,E is measure space .

example (2.1.1). 1. $f(x) = \int_E k(x,t)y(t)dt$ (first kind)

2. $y(x) = f(x) + \lambda \int_E k(x,t)y(t)dt \ , \lambda \in \mathbb{R}.$ (second kind)

2.1.2 Differential equation

Definition 2.1.2. differential equation is an equation where its unknown term is a function , which has a relation between this function and its derivative

$$\frac{d^{(n)}y}{dx^{(n)}} + a_1(x)\frac{d^{(n-1)}y}{dx^{(n-1)}} + ... + a_n(x)y = f(x)$$

where f(x) is giving , with a continuous coefficients $a_1...a_n$ and initial conditions :

$$y(0) = \alpha_0, y_0' = \alpha_1.... ,\ y^{(n+1)}(0) = \alpha_{n-1}$$

example (2.1.2). 1. $xy' + 2y = 4x^2$ (first order)

2. $y^{''} - 3y' + 2y = exp(-x)$ (second order)

2.1.3 Integro-Differential equation

Definition 2.1.3. Let the integro-differential equation

$$y^{(n)}(x) + a_1 y^{(n-1)} + ... + a_n y(x) = f(x) + \int_E k(x,t)y(t)dt$$

The (IDE) is an equation composed of two operators integral and differential
Where :
$a_1......a_n$ are constants
f(x),k(x,t) are giving function
and the unknown function is y(x) .

2.2 Classification of the integro-differential equation

There exists different types of integro differential equation such that we classificate them based on their characteristic in the following 5 types :

1. Integration domain .
2. Kind of equation .
3. The nature of the function f(x) (homogenous).
4. The linearity about the function y(x) .
5. The nature of the kernel and singularity.

We will mention only the first type which is the integration domain (for others types see ([2])).

2.2.1 Fredholm integro-differential equation

The (FIDE) is in the form :

$$y^{(n)}(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt$$

Where the integration domain is fixed .

2.2.2 Volterra integro-differential equation

The (VIDE) is in the form :

$$y^{(n)}(x) = f(x) + \lambda \int_a^x k(x,t)y(t)dt$$

Such that the integration domain is varied with the independent variable in the equation .

2.2.3 Volterra-Frefholm integro-differential equation

The (VFIDE) is combining the both Volterra and Frodholm integral aperator in the same equation in the form :

$$y^{(n)}(x) = f(x) + \int_a^x k_1(x,t)y(t)dt + \int_a^b k(x,t)y(t)dt$$

Remark 2.1. We call the IDE is singular if either one or two the integration limit are infinite , other way the kernel becomes infinite in the voisinage of the integration domain .

2.3 Relationship between Integro-Differential equation and Integral,differential equation

There is interest in reducing the resolution of a Volterra Integro-Differential equation to Volterra Integral equation and vice-versa.

2.3.1 Converting from VIDE to VIE

Lemma 2.1 (Reducing multiple integrals to single integral). *We define the formula which reduce multiple integrals to single integral such as :*

$$\underbrace{\int_a^x \dots \int_a^x}_{n\ times} y(x)dx...dx_n = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1}y(t)dt$$

example (2.3.1). Let the Volterra Integro-Differential equation

$$\begin{cases} y'(x) = 1 - \int_0^x y(t)dt, \\ y(0) = 0. \end{cases}$$

We integer the VIDE

$$\int_0^x y'(x)dt = \int_0^x 1dt - \int_0^x \int_0^x y(t)dtdt$$

$$y(x) - y(0) = x - \int_0^x \int_0^x y(t)dtdt$$

We use the last lemma and the condition
giving a Volterra Integral equation

$$y(x) = x - \int_0^x (x-t)y(t)dt$$

2.3.2 Converting from VIE to VIDE

Lemma 2.2. ***Leibniz Rule***
We define Leibniz rule for Differentiation of Integrals such as :

$$\frac{\partial(\int_{a(x)}^{b(x)} k(x,t)y(t)dt)}{\partial x} = \int_{a(x)}^{b(x)} \frac{\partial k(x,t)}{\partial x} y(t)dt + k(x,b(x))b'(x)y(t) - k(x,a(x))a'(x)y(t)$$

example (2.3.2). Let Volterra Integral equation

$$y(x) = x^3 + \int_0^x (x^2 - t)y(t)dt$$

after the derivative and using Leibniz Rule :

$$\frac{\partial y(x)}{\partial x} = 3x^2 + (x^2 - x)y(x) + 2x \int_0^x y(t)dt.$$

we will obtain the Volterra Integro-Differential equation

$$y'(x) - (x^2 - x)y(x) = 3x^2 - 2x \int_0^x y(t)dt$$

2.3.3 Converting from DE to VIDE

example (2.3.3). Let the Linear Differential equation with the initial condition in point c=0

$$\begin{cases} \frac{\partial^3 y(x)}{\partial x^3} - \frac{\partial y(x)}{\partial x} = \frac{1}{2}x^2, \\ y(0) = 0\ ,\ y'(0) = 0\ ,\ y''(0) = 1. \end{cases}$$

after the following transformation
We integer the DE

$$\int_0^x \frac{\partial^3 y(t)}{\partial x^3} dt - \int_0^x \frac{\partial y(t)}{\partial x} dt = \frac{1}{2}\int_0^x t^2 dt$$

$$\frac{\partial^2 y(x)}{\partial x^2} - \frac{\partial^2 y(0)}{\partial x^2} - y(x) + y(0) = \frac{1}{6}x^3$$

giving

$$\frac{\partial^2 y(x)}{\partial x^2} - y(x) = \frac{1}{6}x^3 + 1 \tag{01}$$

We integer (01)

$$\int_0^x \frac{\partial^2 y(t)}{\partial x^2} dt - \int_0^x y(t)dt = \int_0^x \frac{1}{6}t^3 + 1dt$$

$$\frac{\partial y(x)}{\partial x} - \frac{\partial y(0)}{\partial x} - \int_0^x y(t)dt = \frac{1}{24}x^4 + x$$

giving an IDE of Volterra type

$$y'(x) = \frac{1}{24}x^4 + x + \int_0^x y(t)dt$$

2.4 Analytical solution for Volterra Integro-Differential equation

2.4.1 The Taylor series solution method

We defined the Taylor series in chapter 1

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n$$

For simplicity , the generic form of Taylor series at $x = 0$ can be written as :

$$y(x) = \sum_{n=0}^{N} a_n x^n \tag{2.1}$$

We will apply the series method solution for solving Volterra Integro-Differential equations of the second kind .

example (2.4.1). Let consider the following Volterra Integro-Differential equation :

$$\left\{ \begin{array}{l} y'(x) = 1 + x - x^2 + \int_0^x (x-t)y(t)dt, \;\; 0 < x < 1 \\ y(0) = 3. \end{array} \right. \tag{2.2}$$

Substituting $y(x)$ by the series

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

into both sides of (2.2) leads to :

$$(\sum_{n=0}^{\infty} a_n x^n)' = 1 + x - x^2 + \int_0^x ((x-t)\sum_{n=0}^{\infty} a_n x^n)dt$$

Differentiating the left side once, and by evaluation the integral of the right side
We find

$$\sum_{n=1}^{\infty} n a_n x^{n-1} = 1 + x - x^2 + \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} a_n x^{n+2}$$

Or equivalently

$$\sum_{n=1}^{\infty} (n+1)a_{n+1}x^n = 1 + x - x^2 + \sum_{n=0}^{\infty} \frac{1}{(n-1)(n)} a_{n-2} x^n$$

Where unified the exponent of x in the both side and used $a_0 = 3$ from the given initial condition.
Collecting the coefficients of the same power of x .

$$a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 .... = 1 + x - x^2 + \frac{1}{2}a_0x^2 + \frac{1}{6}a_1x^3$$

$$a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 .... = 1 + x + (-1 + \frac{1}{2}a_0)x^2 + \frac{1}{6}a_1x^3$$

Equating them of like power of x in both side gives:

$$a_0 = 3, a_1 = 1, a_2 = \frac{1}{2}, a_3 = \frac{1}{6}, a_4 = \frac{1}{24}$$

So the series solution is giving by :

$$y(x) = 3 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^3 + ...$$

$$y(x) = 2 + (1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^3 + ...)$$

$$y(x) = 2 + (\sum_{n=0}^{\infty} \frac{1}{n!}x^n)$$

The analytical solution is

$$y(x) = 2 + e^x$$

CHAPTER 3

TAYLOR COLLOCATION METHOD FOR VIDE

In this chapter, we have explained the working instructions of the Taylor collocation method and gave some examples to validate the effectiveness of the method .

3.1 Description of the method

Let us consider the Volterra Integro-Differential equation

$$\sum_{k=0}^{m} p_k(x)y^{(k)}(x) = f(x) + \lambda \int_a^x k(x,t)y(t)dt \tag{001}$$

Where the known function $p_k(x), f(x), k(x,t)$ are defined on the $a \leq x,t \leq b$; λ is a real parameter , y(x) is the unknown function .

FIRSTLY

We substitute the Taylor Collocation point into the equation (1.3) to obtain

$$\sum_{k=0}^{m} p_k(x_i)y^{(k)}(x_i) = f(x_i) + \lambda \int_a^{x_i} k(x_i,t)y(t)dt$$

We put

$$I(x_i) = \int_a^{x_i} k(x_i,t)y(t)dt$$

So that

$$\sum_{k=0}^{m} p_k(x_i)y^{(k)}(x_i) = f(x_i) + \lambda I(x_i) \tag{002}$$

NOW

We can write the system (002) in matrix form

$$\sum_{k=0}^{m} p_k(x_i)y^{(k)}(x_i) = f(x_i) + \lambda I(x_i)$$

$$k = 0 \Longrightarrow p_0(x_i)y^{(0)}(x_i) = f(x_i) + \lambda I(x_i),$$

$$k = 1 \Longrightarrow p_1(x_i)y^{(1)}(x_i) = f(x_i) + \lambda I(x_i),$$

$$\vdots$$

$$k = m \Longrightarrow p_m(x_i)y^{(m)}(x_i) = f(x_i) + \lambda I(x_i).$$

WE TAKE k=0

$$i = 0 \Longrightarrow p_0(x_0)y^{(0)}(x_0) = f(x_0) + \lambda I(x_0),$$

$$i = 1 \Longrightarrow P_0(x_1)y^{(0)}(x_1) = f(x_1) + \lambda I(x_1),$$

$$\vdots$$

$$i = N \Longrightarrow p_0(x_N)y^{(0)}(x_N) = f(x_N) + \lambda I(x_N).$$

The system (002) becomes :

$$\sum_{k=0}^{m} P_k Y^{(k)} = F + \lambda I \tag{003}$$

Where

$$P_k = \begin{pmatrix} p_k(x_0) & 0 & \ldots 0 \\ 0 & p_k(x_1) & \ldots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \ldots p_k(x_k) \end{pmatrix}; Y^{(k)} = \begin{pmatrix} y^0(c) \\ y^1(c) \\ \vdots \\ y^N(c) \end{pmatrix}; F = \begin{pmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_N) \end{pmatrix}; I = \begin{pmatrix} I(x_0) \\ I(x_1) \\ \vdots \\ I(x_N) \end{pmatrix}$$

NOW

Let us assume that the (k th) derivative of (1.1) with respect to x has the truncated taylor series expansion defined by :

$$y^{(k)}(x_i) = \sum_{n=k}^{N} \frac{y^{(n)}(c)}{(n-k)!}(x_i - c)^{n-k}$$

Such that we derivate (1.1) (k th) $k = 0....N$

$$k = 1 \Longrightarrow y'(x_i) = \sum_{n=1}^{N} \frac{ny^{(n)}(c)}{n!}(x_i - c)^{n-1},$$

$$k = 2 \Longrightarrow y''(x_i) = \sum_{n=2}^{N} \frac{n(n-1)y^{(n)}(c)}{n!}(x_i - c)^{n-2},$$

$$\vdots$$

$$k = N \Longrightarrow y^{(N)}(x_i) = \sum_{n=N}^{N} \frac{n(n-1)...(n-N)y^{(n)}(c)}{n!}(x_i - c)^{n-N},$$

We know that n!=n(n-1)(n-2)...
So having

$$y^{(k)}(x_i) = \sum_{n=k}^{N} \frac{y^{(n)}(c)}{(n-k)!}(x_i - c)^{n-k} \tag{004}$$

after that we write (004) in matrix form :

$$k = 0 \Longrightarrow y^{(0)}(x_i) = \frac{y^{(0)}(c)}{0!}(x_i - c)^0 + \frac{y^{(1)}(c)}{1!}(x_i - c)^1.... + \frac{y^{(N)}(c)}{N!}(x_i - c)^N,$$

$$k = 1 \Longrightarrow y^{(1)}(x_i) = \frac{y^{(1)}(c)}{0!}(x_i - c)^0 + \frac{y^{(2)}(c)}{1!}(x_i - c)^1.... + \frac{y^{(N)}(c)}{N-1!}(x_i - c)^{(N-1)},$$

$$k = 2 \Longrightarrow y^{(2)}(x_i) = \frac{y^{(2)}(c)}{0!}(x_i - c)^0 + \frac{y^{(3)}(c)}{1!}(x_i - c)^1.... + \frac{y^{(N)}(c)}{N-2!}(x_i - c)^{(N-2)},$$

$$\vdots$$

$$K = N \Longrightarrow y^{(N)}(x_i) = \frac{y^{(N)}(c)}{0!}(x_i - c)^0 + \frac{Y^{(N+1)}(c)}{1!}(x_i - c)^1... + \frac{y^{(2N)}(c)}{N-N!}(x_i - c)^{(N-N)}.$$

We get

$$Y^{(k)} = XM_kA \tag{005}$$

Then, we substitute the Taylor collocation point in (005)
getting

$$Y^{(k)} = X_{x_i}M_kA$$

Or

$$Y^{(k)} = CM_kA \tag{006}$$

Where

$$C = X_{x_i} = \begin{pmatrix} X_{x_0} \\ X_{x_1} \\ \vdots \\ X_{x_N} \end{pmatrix} = \begin{pmatrix} (x_0-c)^0 & (x_0-c)^1 & \dots (x_0-c)^N \\ (x_1-c)^0 & (x_1-c)^1 & \dots (x_1-c)^N \\ \vdots & \vdots & \ddots \vdots \\ (x_N-c)^0 & (x_N-c)^1 & \dots (x_N-c)^N \end{pmatrix}$$

$$M_k = \begin{pmatrix} 0 & 0 & \dots \frac{1}{0!} & 0 & \dots 0 \\ 0 & 0 & 0 & \frac{1}{1!} & \dots 0 \\ \vdots & \vdots & \ddots \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots 0 & 0 & \dots \frac{1}{(N-k)!} \\ \vdots & \vdots & \dots \vdots & \vdots & \dots \vdots \\ 0 & 0 & \dots 0 & 0 & \dots 0 \end{pmatrix} ; A = \begin{pmatrix} y^0(c) \\ y^1(c) \\ \vdots \\ y^N(c) \end{pmatrix}$$

Now, we can write the equation (003) as a system matrix
Obtaining

$$\sum_{k=0}^{m} P_k C M_k A = F + \lambda I \tag{007}$$

NOW

Let us find the matrix I

$$I(x_i) = \int_a^{x_i} k(x_i, t) y(t) dt \tag{008}$$

Where y(t) can be truncated Taylor series

$$y(t) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(t-c)^n$$

Writing as matrix form so :

$$[y(t)] = T M_0 A \tag{009}$$

The kernel $k(x_i, t)$ is expanded to do truncated Taylor series (in x=c and t=c) in the form :

$$\begin{aligned} k(x_i, t) &= \sum_{n=0}^{N} \sum_{m=0}^{N} \frac{k^{(n)}(c)}{n!}(x_i - c)^n \frac{k^{(m)}(c)}{m!}(t-c)^m \\ &= \sum_{n=0}^{N} \sum_{m=0}^{N} \frac{k^{(n)}(c) k^{(m)}(c)}{n! m!}(x_i - c)^n (t-c)^m \end{aligned}$$

Then

$$\begin{aligned}k_{nm} &= \frac{1}{n!m!}k^{(n)}(c)k^{(m)}(c)\\ &= \frac{1}{n!m!}\frac{\partial^{(n)}k}{\partial x^{(n)}}\frac{\partial^{(m)}k}{\partial t^{(m)}}\\ &= \frac{1}{n!m!}\frac{\partial^{(n+m)}k}{\partial x^{(n)}\partial t^{(m)}}\end{aligned}$$

So $k(x_i, t)$ becomes

$$k(x_i, t) = \sum_{n=0}^{N}\sum_{m=0}^{N} k_{nm}(x_i - c)^n(t - c)^m$$

With the same method we represent the matrix form of k :
We obtain

$$[k(x_i, t)] = CKT^t \tag{010}$$

Where

$$T = \left[(t-c)^0 \;\; (t-c)^1 \;\; \ldots \;\; (t-c)^N\right]$$

$$K = \begin{pmatrix} k_{00} & k_{01} & \ldots k_{0N} \\ k_{10} & k_{11} & \ldots k_{1N} \\ \vdots & \vdots & \ddots \vdots \\ k_{N0} & k_{N1} & \ldots k_{NN} \end{pmatrix}; C = \begin{pmatrix} (x_0-c)^0 & (x_0-c)^1 & \ldots (x_0-c)^N \\ (x_1-c)^0 & (x_1-c)^1 & \ldots (x_1-c)^N \\ \vdots & \vdots & \ddots \vdots \\ (x_N-c)^0 & (x_N-c)^1 & \ldots (x_N-c)^N \end{pmatrix}$$

We substitute (**??**) and (**??**) in the equation (**??**) :

$$I(x_i) = \int_a^{x_i} CKT_tTM_0A dt$$

When CKM_0Adon't depend on t
Therefore

$$I(x_i) = CK\int_a^{x_i} T_tTdtM_0A$$

So

$$\begin{aligned}H &= [h_{nm}]\\ &= \int_a^{x_i} T_tT\\ &= \int_a^{x_i} (t-c)^{n+m}dt \quad n, m = 0, 1...., N\end{aligned}$$

$$= \frac{(x_i - c)^{n+m+1} - (a-c)^{n+m+1}}{n+m+1}$$
$$= H_{x_i}$$

Such that

$$H_{xi} = \begin{pmatrix} h_{00}(x_i) & h_{01}(x_i) & \dots h_{0N}(x_i) \\ h_{10}(x_i) & h_{11}(x_i) & \dots h_{1N}(x_i) \\ \vdots & \vdots & \vdots\vdots \\ h_{N0}(x_i) & h_{N1}(x_i) & \dots h_{NN}(x_i) \end{pmatrix}$$

Hence , we obtain the matrix I as

$$I = \overline{XKHM_0}A \tag{011}$$

Where $\overline{X}$, $\overline{K}$, $\overline{H}$ and $\overline{M_0}$ respectively $(N+1)by(N+1)^2$, $(N+1)^2by(N+1)^2$, $(N+1)^2by(N+1)^2$ and $(N+1)^2by(N+1)$ can be written by the blocked matrices as follows :

$$\overline{X} = \begin{pmatrix} X_{x_0} & 0 & \dots 0 \\ 0 & X_{x_1} & \dots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots X_{x_N} \end{pmatrix}, \overline{K} = \begin{pmatrix} K & 0 & \dots 0 \\ 0 & K & \dots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots K \end{pmatrix}$$

$$\overline{H} = \begin{pmatrix} H_{x_0} & 0 & \dots 0 \\ 0 & H_{x_1} & \dots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots H_{x_N} \end{pmatrix}, \overline{M_0} = \begin{pmatrix} M_0 \\ M_0 \\ \vdots \\ M_0 \end{pmatrix}$$

FINALLY
We substitute (**??**) into (**??**)
getting

$$\sum_{k=0}^{m} P_k C M_k A = F + \lambda \overline{XKHM_0} A$$
$$\sum_{k=0}^{m} P_k C M_k A - \lambda \overline{XKHM_0} A = F$$
$$(\sum_{k=0}^{m} P_k C M_k - \lambda \overline{XKHM_0}) A = F \tag{012}$$

This is the system algebraic of (N+1) equation with the unknown Taylor coefficients and the fundamental matrix for Volterra integro-differential equation

Therefore, we can write (012) in the form

$$WA = F \tag{013}$$

Where

$$W = [W_{ij}] = \sum_{k=0}^{m} P_k C M_k - \lambda \overline{XKHM} \quad i, j = 0, ..., N$$

Then we can find the unknown coefficients by means of the augmented matrix of (013)

$$[W; F] = \begin{bmatrix} W_{00} & W_{01} & \dots W_{0N} & ; f(x_0) \\ W_{10} & W_{11} & \dots W_{1N} & ; f(x_1) \\ \vdots & \vdots & \ddots \vdots & ; \vdots \\ W_{N0} & W_{N1} & \dots W_{NN} & ; f(x_N) \end{bmatrix} \tag{014}$$

If $|W| \neq 0$ in (013) we get

$$A = W^{-1} F \tag{015}$$

Thus, the unknown coefficients are uniquely determined by (015) and thereby we find any particular solution of Volterra integro-differential equation in the truncated Taylor series . Else, the equation either is insoluble or has an infinite number of solution .

NOW

Let us form the matrix representation of the condition

In the most general

$$\sum_{j=0}^{m-1} a_{ij} y^{(j)}(a) + b_{ij} y^{(j)}(b) + c_{ij} y^{(j)}(c) = \lambda_i \quad i = 0, 1....., m-1 \tag{016}$$

Where $a \leq c \leq b$, a_{ij}, b_{ij}, c_{ij} and λ_i are constants
and the functions at the point a , b and c are expressed in the truncated Taylor series
Such that finding

$$[y^{(j)}(a)] = PM_j A \tag{017}$$

$$[y^{(j)}(b)] = QM_j A \tag{018}$$

$$[y^{(j)}(c)] = RM_j A \tag{019}$$

Where

$$P = [1 \ \ (a-c) \ \ (a-c)^2 \ \ \ldots \ \ (a_c)^N]$$

$$Q = [1 \ \ (b-c) \ \ (b-c)^2 \ \ \ldots \ \ (b-c)^N]$$

$$R = [1 \ \ 0 \ \ 0 \ \ \ldots \ \ 0]$$

Substituting the matrix (017),(018) and (019) into the equation (016)
We obtain

$$\sum_{j=0}^{m-1}(a_{ij}P + b_{ij}Q + c_{ij}R)M_j A = [\lambda_i]$$

We put

$$U_i = \sum_{j=0}^{m-1}(a_{ij}P + b_{ij}Q + c_{ij}R)M_j \equiv [u_{i0} \ \ u_{i1} \ \ \ldots \ \ u_{iN}] \ \ i = 0, 1, ..., m-1$$

Thus , the matrix form conditions (016) become

$$U_i A = [\lambda_i] \tag{020}$$

With the augmented matrices

$$[U_i; \lambda_i] = [u_{i0} \ \ u_{i1} \ \ \ldots \ \ u_{iN}; \lambda_i] \tag{021}$$

CONSEQUENTLY, replacing the m row matrices of (021) by the last m row of the augmented matrix (014) , we have the required augmented matrix

$$[\widetilde{W}; \widetilde{F}] \tag{022}$$

where

$$\widetilde{W} = \begin{bmatrix} w_{00} & w_{01} & \ldots w_{0N} \\ w_{10} & w_{11} & \ldots w_{1N} \\ \vdots & \vdots & \ddots \vdots \\ w_{N-m,0} & w_{N-m,1} & \ldots w_{N-m,N} \\ u_{00} & u_{01} & \ldots u_{0N} \\ \vdots & \vdots & \ddots \vdots \\ u_{m-1,0} & u_{m-1,1} & \ldots u_{m-1,N} \end{bmatrix}; \widetilde{F} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_{N-m}) \\ \lambda_0 \\ \vdots \\ \lambda_{m-1} \end{bmatrix}$$

If $\left|\widetilde{W}\right| \neq 0$ we can write

$$A = \widetilde{W}^{-1}\widetilde{F} \tag{3.1}$$

Thus, the matrix A is uniquely determined ,then we can say that the Volterra integro-differential equation (001) with the conditions (016) has a unique solution in the form (1.1).

3.2 Illustration

We give some examples to illustrate the use of the method of Taylor collocation into Volterra integro-differential equation

example (3.2.1). Let us consider the Volterra integro-differential equation of the second order given by :

$$\begin{cases} y'(x) + y(x) = (1 + 2x) + \int_0^x x(1 + 2x)e^{(t(x-t))}y(t)dt, \quad 0 \le x \le 1 \\ y(0) = 1. \end{cases}$$

Where

$a = 0; b = 1; c = 0; \lambda = 1$

$p_1 = 1; p_0 = 1; f(x) = 1 + 2x; k(x,t) = x(1 + 2x)e^{(t(x-t))}$

and the approximation the solution $y(x)$ by the Taylor polynomial

$$y(x) = \sum_{n=0}^{3} \frac{y^{(n)}(0)}{n!}x^n$$

The exact solution $y(x) = e^{x^2}$ Then , for $N = 3$ the collocation points :

$$x_0 = 0; x_1 = \frac{1}{3}; x_2 = \frac{2}{3}; x_3 = 1$$

The matrix (012) :

$$(P_1CM_1 + P_0CM_0 - \overline{XKHM_0})A = F$$

such that P_1,C,M_1,P_0 and M_0 are matrices of order (4×4) defined by

$$P_1 = P_0 = \begin{pmatrix} 1 & 0 & \dots 0 \\ 0 & 1 & \dots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots 1 \end{pmatrix}; C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} \\ 1 & \frac{2}{3} & \frac{4}{9} & \frac{8}{27} \\ 1 & 1 & 1 & 1 \end{pmatrix}; M_1 = \begin{pmatrix} 0 & \frac{1}{0!} & 0 & 0 \\ 0 & 0 & \frac{1}{1!} & 0 \\ 0 & 0 & 0 & \frac{1}{2!} \\ 0 & 0 & 0 & 0 \end{pmatrix}; M_0 = \begin{pmatrix} \frac{1}{0!} & 0 & 0 & 0 \\ 0 & \frac{1}{1!} & 0 & 0 \\ 0 & 0 & \frac{1}{2!} & 0 \\ 0 & 0 & 0 & \frac{1}{3!} \end{pmatrix}.$$

and $\overline{X}$ is matrix of order (4×16) defined by :

$$\overline{X} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{2}{3} & \frac{4}{9} & \frac{8}{27} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

and $\overline{K}, \overline{H}$ are matrices of order (16×16)

Where

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 1 & -2 & -1 \\ 0 & 2 & \frac{1}{2} & -2 \end{pmatrix} \Longrightarrow \overline{K} = \begin{pmatrix} K & 0 & 0 & 0 \\ 0 & K & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{pmatrix}.$$

$$H_{x_i} = \begin{pmatrix} x_i & \frac{x_i^2}{2} & \frac{x_i^3}{3} & \frac{x_i^4}{4} \\ \frac{x_i^2}{2} & \frac{x_i^3}{3} & \frac{x_i^4}{4} & \frac{x_i^5}{5} \\ \frac{x_i^3}{3} & \frac{x_i^4}{4} & \frac{x_i^5}{5} & \frac{x_i^6}{6} \\ \frac{x_i^4}{4} & \frac{x_i^5}{5} & \frac{x_i^6}{6} & \frac{x_i^7}{7} \end{pmatrix}. \Longrightarrow \overline{K} = \begin{pmatrix} H_{x_0} & 0 & 0 & 0 \\ 0 & H_{x_1} & 0 & 0 \\ 0 & 0 & H_{x_2} & 0 \\ 0 & 0 & 0 & H_{x_3} \end{pmatrix}.$$

And the order of $\overline{M_0}$ is (16×4) such that

$$\overline{M_0} = \begin{pmatrix} M_0 \\ M_0 \\ M_0 \\ M_0 \end{pmatrix}; F = \begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ f(x_3) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5}{3} \\ \frac{7}{3} \\ 3 \end{pmatrix}.$$

After the augmented matrices of the system and condition are computed ,

We obtain the new augmented matrix in the form

$$[\widetilde{W}; \widetilde{F}] = \begin{bmatrix} 1 & 1 & 0 & 0 & ;1 \\ \frac{263}{324} & \frac{2548}{1957} & \frac{5407}{14029} & \frac{134}{2181} & ;\frac{5}{3} \\ \frac{-169}{2187} & \frac{1685}{1281} & \frac{594}{731} & \frac{291}{1123} & ;\frac{7}{3} \\ 1 & 0 & 0 & 0 & ;0 \end{bmatrix}$$

This system has the solution

$$A = [1 \;\; 0 \;\; 1.2951 \;\; 1.9323]$$

Therefor, we find the solution

$$y(x) = 1 + 0.64754x^2 + 0.966173x^3 \cong y(x) = e^{x^2}$$

CHAPTER 4

COMPARISON BETWEEN THE EXACT AND THE APPROACH SOLUTION

In the last chapter, we are presented three examples to illustrate the accuracy and efficiently of the method, and the result are compared with the exact solution , the tables and figures are provided the validity and the applicability of the method .

4.1 Example01

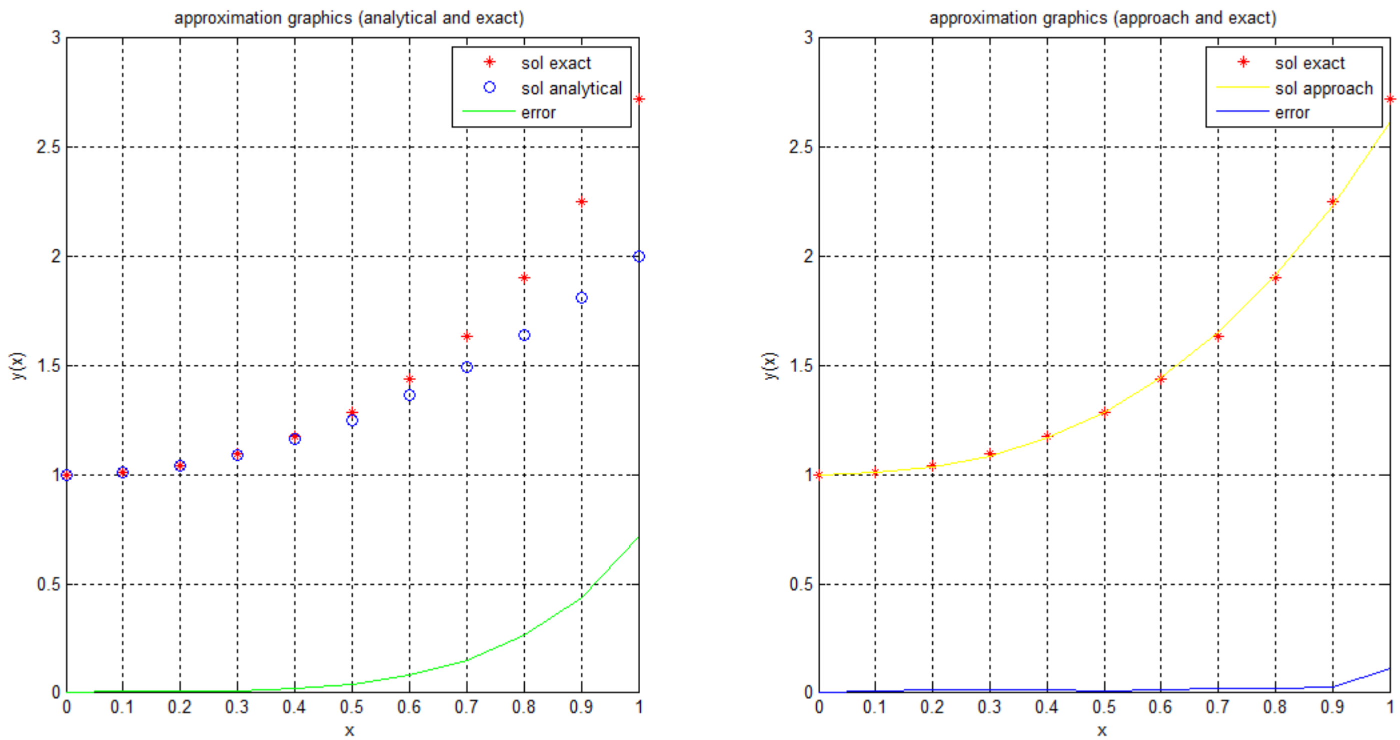
Let us consider the LVIDE

$$\begin{cases} y'(x) + y(x) = 1 + 2x + \int_0^x x(1+2x)e^{t(x-t)}y(t)dt, & 0 \le x \le 1 \\ y(0) = 0. \end{cases}$$

The exact solution is $y(x) = e^{x^2}$

x	sol(ex)	sol(app)(N=3)	error1	sol(ana)(N=3)	error2
0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
0.1000	1.0101	1.0074	2.6086e-03	1.0100	5.0167e-05
0.2000	1.0408	1.0336	7.1798e-03	1.0400	8.1077e-04
0.3000	1.0942	1.0844	9.8090e-03	1.0900	4.1743e-03
0.4000	1.1735	1.1654	8.0694e-03	1.1600	1.3511e-02
0.5000	1.2840	1.2827	1.3688e-03	1.2500	3.4025e-02
0.6000	1.4333	1.4418	8.4784e-03	1.3600	7.3329e-02
0.7000	1.6323	1.6487	1.6376e-02	1.4900	1.4232e-01
0.8000	1.8965	1.9091	1.2625e-02	1.6400	2.5648e-01
0.9000	2.2479	2.2288	1.9060e-02	1.8100	4.3791e-01
1.0000	2.7183	2.6137	1.0457e-01	2.0000	7.1828e-01

$TableI$



for N=3

4.2 Example02

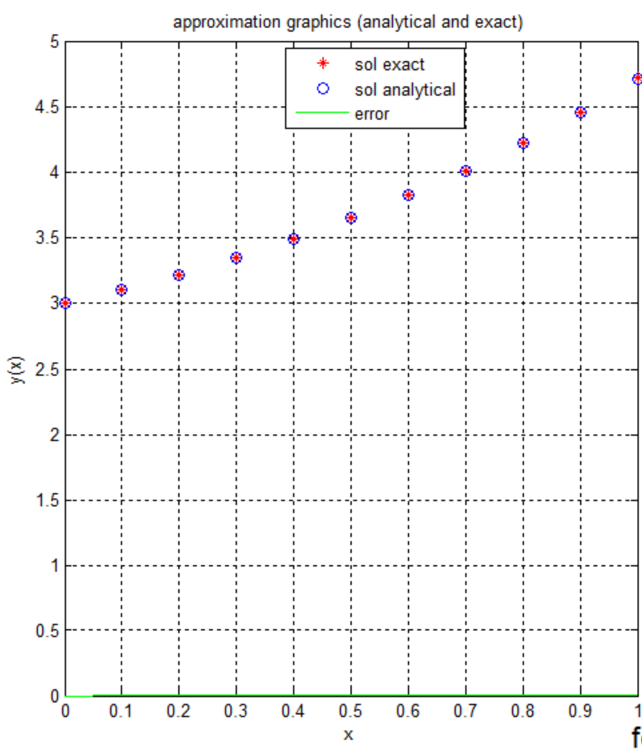
Let us consider the VIDE

$$\begin{cases} y'(x) = 1 + x - x^2 + \int_0^x (x-t)y(t)dt, \\ y(0) = 3. \end{cases}$$

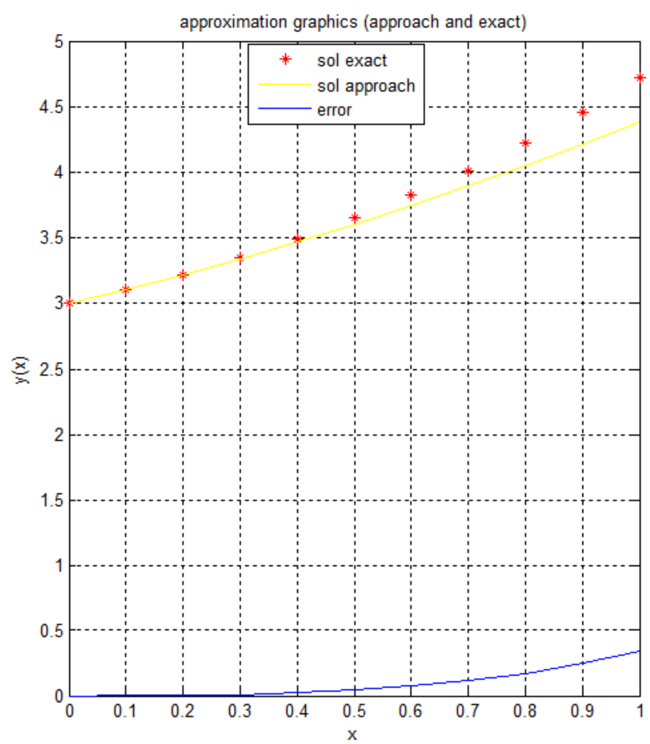
where the exact solution is $y(x) = 2 + e^x$
and the solution obtained are compared with the exact solution in Table II .

x	sol(ex)	sol(app)(N=5)	error1	sol(ana)(N=5)	error2
0.0000	3.0000	3.0000	0.0000	3.0000	0.0000
0.1000	3.1052	3.1048	3.3721e-04	3.1052	8.4742e-08
0.2000	3.2214	3.2187	2.6919e-03	3.2214	2.7582e-06
0.3000	3.3499	3.3408	9.0817e-03	3.3498	2.1308e-05
0.4000	3.4918	3.4703	2.1526e-02	3.4917	9.1364e-05
0.5000	3.6487	3.6067	4.2043e-02	3.6484	2.8377e-04
0.6000	3.8221	3.7495	7.2651e-02	3.8215	7.1880e-04
0.7000	4.0138	3.8984	1.1537e-01	4.0122	1.5819e-03
0.8000	4.2255	4.0533	1.7221e-01	4.2224	3.1409e-03
0.9000	4.4596	4.2144	2.4520e-01	4.4538	5.7656e-03
1.0000	4.7183	4.3819	3.3638e-01	4.7083	9.9485e-03

$TableII$



for N=5



4.3 Example03

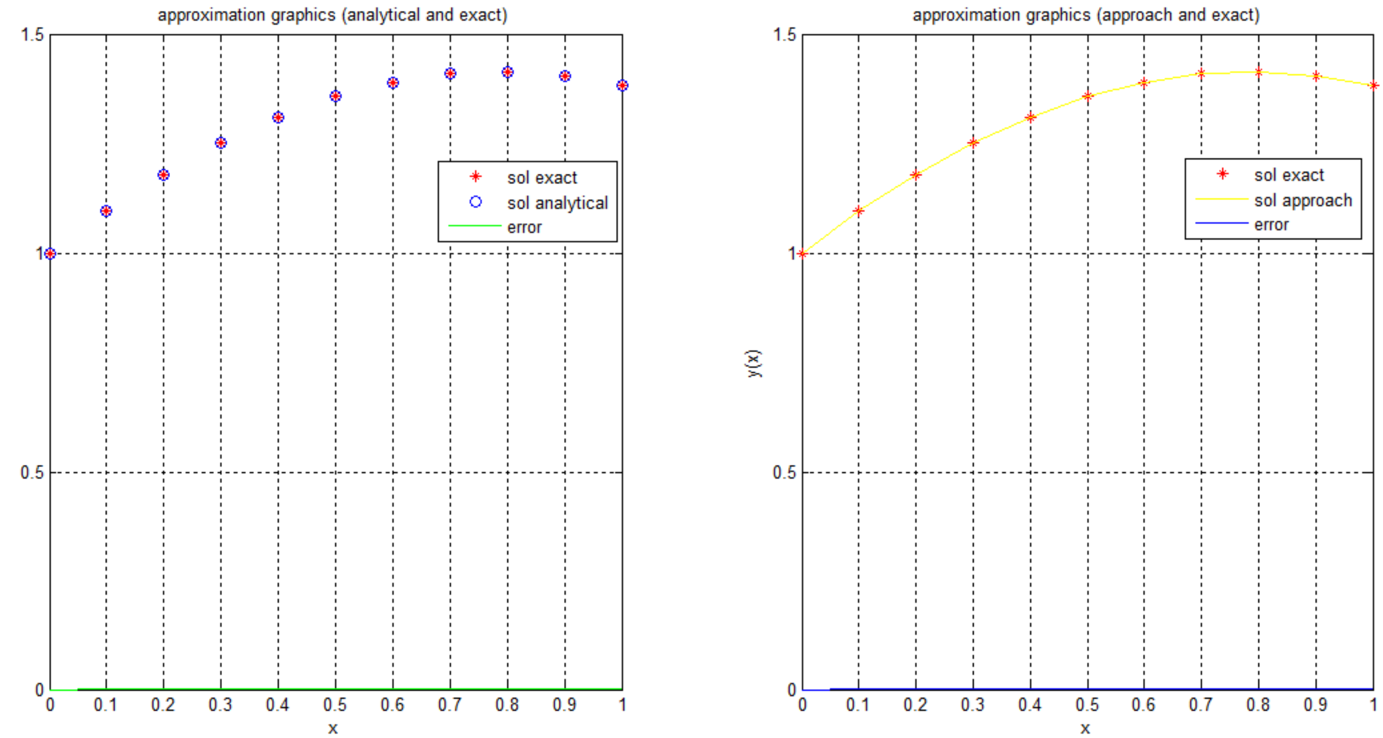
Let the following volterra Integro-Differential Equation

$$\begin{cases} y''(x) = -1 - x + \int_0^x (x-t)y(t)dt, \\ y(0) = 1, \\ y'(0) = 1. \end{cases}$$

$y(x) = sin(x) + cos(x)$ is the exact solution
the comparison in the table III .

x	sol(ex)	sol(app)(N=4)	error1	sol(app)(N=8)	error2
0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
0.1000	1.0948	1.0948	2.9426e-08	1.0948	1.9591e-11
0.2000	1.1787	1.1787	1.9689e-07	1.1787	2.4748e-09
0.3000	1.2509	1.2509	5.1385e-07	1.2509	4.1713e-08
0.4000	1.3105	1.3105	8.0985e-07	1.3105	3.0813e-07
0.5000	1.3570	1.3570	6.7862e-07	1.3570	1.4481e-06
0.6000	1.3900	1.3900	5.5208e-07	1.3900	5.1117e-06
0.7000	1.4091	1.4091	3.7452e-06	1.4091	1.4807e-05
0.8000	1.4141	1.4141	9.6385e-06	1.4141	3.7111e-05
0.9000	1.4049	1.4050	1.8100e-05	1.4050	8.3260e-05
1.0000	1.3818	1.3818	2.6709e-05	1.3819	1.7115e-04

$TableIII$


approximation graphics (analytical and exact)
sol exact
sol analytical
error
x
approximation graphics (approach and exact)
sol exact
sol approach
error
y(x)
x

for N=7

General Conclusion

In this work, we applied the Taylor Collocation method to the Volterra Integro-Differential equation to find the approximate solution.
We found that the method gives an approximate converge toward the exact solution through examples .
For the exponential (e) and trigonometric$(sin(x), cos(x))$ functions , we saw that whenever (n) is augmented ,whenever the approximate solution converge toward the exact solution .
From this we conclude that Taylor Collocation is applicable to the Volterra Integro-Differential equation .

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الملخص : في هذه المذكرة ,قمنا بحل معادلة خطية تكاملية-تفاضلية من نوع فولتيرا الصنف الثاني باستعمال طريقة تايلور التقطيعية التي تحول المعادلة إلى برنامج جبري خطي . دعمنا بأمثلة من اجل إثبات صحة الطريقة .

الكلمات المفتاحية : معادلة خطية تكاملية-تفاضلية من نوع فولتيرا الصنف الثاني, طريقة تايلور التقطيعية.

Dans ce mémoire, nous avons résolu l'equation Integro-Differentiel linéaire de Volterra de second espèce numériquement , en utilisant la méthode de Collocation de Taylor qui convertit l'équation en système linéaire algebrique. Nous avons étayé par des exemples pour prouver la validité de la méthode .

Mots-Clés: Equation Integro-Differentiel linéaire de Volterra de second espèce ,méthode de Collocation de Taylor.

In this thesis, we have solved the linear Volterra Integro-Differential equation od the second kind numerically,using the Taylor Collocation method which converts the equation into an algebraic linear system . We have strengthened by examples for proved the accuracy of the method.

Keywords:
Volterra linear Integro-Differential equation of the second kind , Taylor Collocation method .