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A unified framework for the performance evaluation of single-branch dual-hop AF relaying in the presence of transceiver hardware impairments

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In this paper, a general four-parameter end-to-end signal-to-noise with distortion ratio (SNDR) model for single-branch dual-hop amplify-and-forward (AF) relaying with/without the presence of transceiver hardware impairments is proposed. This model encompasses standard as well as non-standard AF relay configurations for both ideal and non-ideal transceiver hardware cases. A unified framework to evaluate the performance of single-branch dual-hop AF relaying is then formulated. This is accomplished by deriving new closed-form expressions for some statistics of some functions of two independent Gamma random variables. Based on these closed-form expressions, an exact analytical expression for the outage probability (OP) and an upper bound for the ergodic capacity are derived. Monte Carlo simulations' results are provided to verify the accuracy of the analytical results.

1. Introduction

Cooperative relaying protocols can be roughly categorized as regenerative and non-regenerative relaying. Using the family of regenerative relaying, the relay modifies the signal hence some digital baseband operations are required [1]. A prominent example of regenerative relaying is decode-and-forward (DF) relaying [2,3], where the relay detects and decodes the signal before forwarding it to the destination. In the case of non-regenerative relaying, the relay does not modify the signal, but it just performs very simple operations such as amplification, phase rotation, etc. [1]. Amplify-and-forward (AF) relaying [4,5], where the relay simply amplifies the signal that it receives, represents the simplest and most popular non-regenerative protocol.

For AF relaying, an important design issue is the choice of the amplification factor based on the available source-to-relay channel state-information (CSI) and the noise statistics. As standard AF relay configurations, we have channel-noise-assisted (CNA) relaying [6,7], channel-assisted (CA) relaying [8] and blind relaying [9,10].

To combat fading introduced by multi-path propagation, in a wireless network, new low-complexity cooperative diversity protocols were introduced in [11]. In [12], authors exploited node co-operation to develop DF and compress-and-forward (CF) strategies for relay networks with many relays, antennas, sources and destinations. The DF relay strategy has attracted authors in [13] to develop a low-complexity coherent demodulator, at the destination node, in the form of a weighted combiner where the weight can be selected adaptively to account for the quality of both the source-to-relay (SR) and the source-to-destination (SD) links. An optical dual-hop single-branch AF relaying scheme in an asymmetric RF-FSO environment, where the RF link experiences Nakagami-m fading while the FSO link experiences Gamma–Gamma turbulence is considered in [14]. Closed-form expressions for the outage probability (OP), the average bit-error-rate (BER) and the average channel capacity were provided. In an asymmetric $\eta-\mu/k-\mu$ fading environment, reference [3], studied the end-to-end (E2E) performance of a dual-hop single-branch DF relaying scheme. Recently, authors of [15–17], have conducted interesting researches on multiple-access (MA) techniques in multi-branch dual-hop relaying under some symmetric/asymmetric fading scenarios.

Ref. [18] provided a unified model for the analysis of single-branch dual-hop AF relaying over Nakagami-m faded links. It gave exact analytical expressions for the cumulative-distribution function (CDF), the probability density function (PDF), and the moment-generating function (MGF) of the end-to-end signal-to-noise ratio (SNR). However, like most of the works on cooperative relaying, this model assumes an ideal transceiver hardware. In practice, hardware experiences a variety of impairments, for instance, I/Q imbalance, phase noise and high power amplifier

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(HPA) non-linearities along with other types of impairments [19–21]. Great efforts have been made in [22,23] to analyze the effects of hardware impairments on different schemes of single-hop systems, For example, authors of [24] have considered the impact of I/Q imbalance on zero-IF MIMO OFDM transceivers. They have showed that it creates a symbol error rate (SER) floor as a result of creating an additional image-signal from the minor sub-carrier. Furthermore, it attenuates the amplitude and rotates the phase of the desired constellation. Besides, the effect of non-linear HPAs has been characterized as a distortion of the constellation position plus an additive Gaussian noise in [25]. Although many algorithms to compensate the impairments in hardware were developed, there are still residual impairments [20–22]. From the results of the works in [22,23], one can conclude that hardware impairments have mischievous effects on the performance of the systems in consideration. In [20], the author showed that high rate wireless systems are more affected by hardware impairments and this is more remarkable when considering low-cost hardware. A study in [26], on the capacity limits of MIMO channels in the presence of transceiver impairments, has shown that multi-antennas systems can be gravely affected by hardware impairments and that a finite capacity limit appears at high signal-to-noise ratio (SNR). A framework for optimal resource allocation to design the existing signal processing algorithms to take into account hardware impairment was successfully developed in [23].

In spite of the detrimental effect of hardware impairments on the performance of cooperative relaying, most of the previous works on signal-branch dual-hop AF relaying have not investigated their impact for widely used relay configurations. To the best of our knowledge, [27] is the first paper presenting an analytical study of single-branch dual-hop AF relaying in the presence of transceiver hardware impairments and considering the generalized model of [20,28], where two CNA relay configurations, fixed and variable gain relaying, were considered. Unfortunately, the framework in [27] cannot handle the outage probability (OP) and the ergodic capacity for other standard and non-standard (NS) AF relay configurations. This paper makes the following specific contributions:

- We introduce a generalized E2E signal-to-noise with distortion ratio (SNDR) model that unlike [18] accounts for transceiver hardware impairments and unlike [27] covers all possible standard as well as NS AF relay configurations.
- After proposing the E2E SNDR model, we have derived Theorem 1 that will be used latter to formulate a new closed-form expression for the OP of the system in consideration. By this expression, we are able to characterize the impact of hardware impairment for any arbitrary SNR value. Theorems 2, 3 and Corollary 1 will be then used to provide a new upper bound on the ergodic capacity. Note that, in our model the channel magnitude of each one of the two hops can be modeled either as a Rayleigh or a Nakagami-m variate, so four different link1/link2 fading scenarios (two of them are symmetric: Nakagami-m/Nakagami-m and Rayleigh/Rayleigh while the two others are asymmetric: Nakagami-m/Rayleigh and Rayleigh/Nakagami-m) can be covered.

Our contribution aims to extend the contributions in [18] and [27] to provide a unified mathematical framework, for the performance evaluation of single-branch dual-hop AF relayed transmission, to handle both ideal and non-ideal hardware cases and to cover all possible standard (i.e., CNA, CA and blind) as well as NS AF relay configurations.

The remainder of this paper is organized as follows. The system model is presented in the next section. Key results for some statistics of some functions of two independent Gamma distributed random variables (RVs) are presented in Section 3. These results are applied in Section 4 to evaluate the performance of single-branch dual-hop AF relaying. Finally, Section 5 concludes the paper.

Here we give, for quick reference, the common notations used in this paper. $K_v(.)$ is the modified Bessel function of the second kind [29, Eq. (9.6)] of order v. $W_{\mu,v}(.)$ denotes the Whittaker function [29, Eq. (13.1)]. $U(x)$ is the unit step function. $f_X(x)$, $F_X(x)$, $\bar{F}_X(x)$ and $M_X(s)$ denote, respectively, the PDF, the CDF, the complementary CDF and the MGF of a continuous RV X. Z^+ denotes the set of positive integers. The gamma function $\Gamma(n)$ of an integer n satisfies $\Gamma(n) = (n-1)!$.

2. System and channel model

This work considers a single-branch dual-hop AF relayed path $S \rightarrow R \rightarrow D$, where a source communicates with a destination through a relay; see Fig. 1. As in [18,27,30], the direct path and other paths that may lead from the source to the destination are not considered here. A distortion noise η_i (introduced in [27, section II-A]) originating from transceiver hardware impairments acting as an unknown noise-like interfering signal is considered. For the E2E SNDR, Λ of this system, we propose the following generalized model

$$\Lambda = \frac{\Lambda_1\Lambda_2}{d\Lambda_1\Lambda_2 + b_1\Lambda_1 + b_2\Lambda_2 + c}, \tag{1}$$

where $\Lambda_i = |h_i|^2$ represents the channel for the ith hop. The channel magnitudes $\{|h_i|\}$ are modeled as independent but not necessarily identically distributed Nakagami-m RVs. The E2E SNDR in (1) corresponds to a generalization of the relay gain in [27, Eq. (12)] and that in [18] to be $G^2 = P_2/(P_1(a + k_1^2) + bN_1)$. The strictly positive parameters b_1, b_2, c and d in (1) are given by

$$\begin{cases} b_1 = (a + k_1^2)\frac{N_2}{P_2} \\ b_2 = (1 + bk_2^2)\frac{N_1}{P_1} \\ c = b\frac{N_1}{P_1}\frac{N_2}{P_2} \\ d = k_1^2 + k_2^2(a + k_1^2) \end{cases} \tag{2}$$

where $P_i = E(|s_i|^2)$ and $N_i = E(|n_i|^2)$ are the average powers for the transmitted signal s_i and the added noise n_i (at the ith hop), respectively. The parameter $k_i > 0$ describes the level of impairment on the i^{th} hop [27,31]. $k_1 = k_2 = 0$ corresponds to the ideal hardware model. Setting $k_1 = k_2 = 0$, $P_1 = P_2 = 1$, and $N_1 = N_2 = 1$ corresponds to the model considered in [18] (i.e., ideal hardware with normalized source and relay powers).

Note that the choice of the values of parameters a and b reflects the relay configuration, i.e., how to use the source-to-relay CSI and the noise statistics to determine the relay functioning (standard configurations, namely, blind, CA, and CNA are represented, respectively, with $(a, b) \in \{(0, c), (1, 0), (1, 1)\}$, where c is a positive constant. Other values (e.g., $a = 0.5$, $b = 1$) represent NS configurations).

3. Theory

In this section, we first recall the definitions of Gamma and Exponential RVs, then we present some key results that will be used later in Section 4 to evaluate the performance of dual-hop AF relayed transmission.

3.1. Definitions

Definition 1 (*Gamma RV*). X follows a Gamma distribution with a shape parameter $\alpha \in Z^+$, and a scale parameter $\beta > 0$ if its PDF is given by

$$f_X(x; \alpha, \beta) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha}U(x). \tag{3}$$

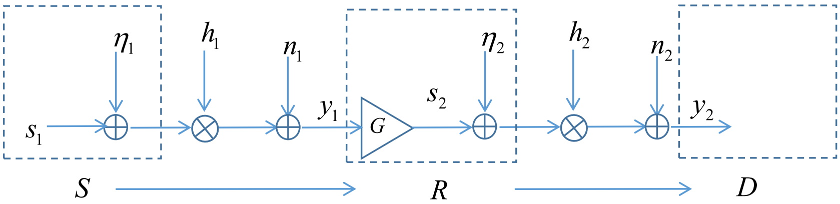


Fig. 1. Block diagram of single-branch dual-hop AF relaying (with transceiver hardware impairments).

In what follows, the notation $X \sim Gamma(\alpha, \beta)$ will be used to denote that X is a Gamma distributed RV with parameters α and β.

Definition 2 (*Exponential RV*). *X follows an Exponential distribution with a rate parameter $\gamma > 0$ if its PDF is given by*

$$f_X(x;\gamma) = \gamma e^{-\gamma x} U(x). \tag{4}$$

In what follows, the notation $X \sim Exp(\gamma)$ will be used to denote that X is an Exponential distributed RV with parameter γ. Note that (3) reduces to (4) when $1/\beta = \gamma$ and $\alpha = 1$.

3.2. Main results

Theorem 1 (*CDF of $X = X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d)$*). *Let X_1 and X_2 be two independent Gamma RVs with parameters (α_1, β_1) and (α_2, β_2) respectively, [i.e., $X_i \sim Gamma(\alpha_i, \beta_i)$, $i = 1, 2$]. Let a, b, c, and d be strictly positive constants. Then, the CDF of the RV $X = X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d)$, $F_X(x)$, is given by*

$$\begin{aligned} F_X(x) = {} & 1 - 2e^{-\left(\frac{c}{\beta_1}+\frac{b}{\beta_2}\right)\left(\frac{x}{1-ax}\right)} \\ & \times \sum_{j=0}^{\alpha_1-1}\sum_{n=0}^{\alpha_2-1}\sum_{k=0}^{j} c_1(j,n,k)\left(\frac{x}{1-ax}\right)^{\alpha_2+j} \\ & \times \left(bc + \frac{d(1-ax)}{x}\right)^{\frac{n+k+1}{2}} \\ & \times K_{n-k+1}\left(2\sqrt{\frac{bcx^2}{\beta_1\beta_2(1-ax)^2} + \frac{dx}{\beta_1\beta_2(1-ax)}}\right) \end{aligned} \tag{5}$$

for $x < \frac{1}{a}$ and $F_X(x) = 1$ for $x \geq \frac{1}{a}$, where $c_1(j,n,k) = \frac{b^{\alpha_2-n-1}c^{j-k}\beta_1^{\frac{k-n-1-2j}{2}}\beta_2^{\frac{n-k+1-2\alpha_2}{2}}}{k!(j-k)!n!(\alpha_2-n-1)!}$.

Proof. See the Appendix. ■

In the special case when $X_i \sim Exp(\gamma_i)$, $i = 1, 2$, the CDF in (5) reduces to

$$\begin{aligned} F_X(x) = {} & 1 - 2\sqrt{\gamma_1\gamma_2}e^{(c\gamma_1+b\gamma_2)\left(\frac{x}{1-ax}\right)}\sqrt{\frac{bcx^2}{(1-ax)^2} + \frac{dx}{(1-ax)}} \\ & \times K_1\left(2\sqrt{\gamma_1\gamma_2\left(\frac{bcx^2}{(1-ax)^2} + \frac{dx}{(1-ax)}\right)}\right) \end{aligned} \tag{6}$$

for $x < \frac{1}{a}$ and $F_X(x) = 1$ for $x \geq \frac{1}{a}$.

Theorem 2 (*MGF of $X = X_1X_2/(bX_1 + cX_2 + d)$*). *Let X_1 and X_2 be two independent Gamma RVs with parameters (α_1, β_1) and (α_2, β_2) respectively, [i.e., $X_i \sim Gamma(\alpha_i, \beta_i)$, $i = 1, 2$]. Let $b, d \geq 0$ and $c > 0$. Then, the MGF of the RV $X = X_1X_2/(bX_1 + cX_2 + d)$, $M_X(s) = E_X\{e^{-sX}\}$, is given by the following.*

Case($b \neq 0$, $d \neq 0$):

$$\begin{aligned} M_X(s) = {} & 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_2(n,k,m)(bc)^{\frac{n+m+1}{2}} \\ & \times \sum_{q=0}^{n+m+2}\binom{n+m+2}{q} \\ & \times \left(\frac{d}{bc}\right)^q J_1(n,k,m,q) \end{aligned} \tag{7}$$

where $c_2(n,k,m) = \frac{b^{k-m}c^{\alpha_1-n-1}\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{m!(k-m)!n!(\alpha_1-n-1)!}$, and

$$\begin{aligned} & J_1(n,k,m,q) \\ & = \frac{\sqrt{bc\beta_1\beta_2}\Gamma(n+2)\Gamma(m+1)}{2d(-1)^{\alpha_1+k-q+1}}\frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}}\left\{e^{\frac{dp}{2bc}}\right. \\ & \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{d(p+\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}})}{2bc}\right) \\ & \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \\ & \times \left.\left(\frac{d(p-\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}})}{2bc}\right)\right\}_{|p=s+\frac{c}{\beta_1}+\frac{b}{\beta_2}}. \end{aligned}$$

Case($b \neq 0$, $d = 0$):

$$M_X(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)(bc)^{\frac{k+m+1}{2}}J_2(n,k,m) \tag{8}$$

where,

$$\begin{aligned} & J_2(n,k,m) \\ & = \frac{\sqrt{\pi}\Gamma(\alpha_1+k+n-m+2)\Gamma(\alpha_1-k+n+m)}{\Gamma(\alpha_1+n+3/2)} \\ & \times \left(\frac{16bc}{\beta_1\beta_2}\right)^{\frac{k-m+1}{2}} \\ & \times \frac{{}_2F_1(\alpha_1+k+n-m+2; k-m+3/2; \alpha_1+n+3/2; \bar{s})}{\left(s+\left(\sqrt{\frac{c}{\beta_1}}+\sqrt{\frac{b}{\beta_2}}\right)^2\right)^{\alpha_1+k+n-m+2}}. \end{aligned}$$

with, $\bar{s} = \left(s+\left(\sqrt{\frac{c}{\beta_1}}-\sqrt{\frac{b}{\beta_2}}\right)^2\right)/\left(s+\left(\sqrt{\frac{c}{\beta_1}}+\sqrt{\frac{b}{\beta_2}}\right)^2\right)$.

Case($b = 0$, $d \neq 0$):

$$M_X(s) = 1 - 2s\sum_{q=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} c_2(q,k)J_2(q,k) \tag{9}$$

where, $c_2(q,k) = \frac{\beta_1^{\frac{q-k+1-2\alpha_1}{2}}\left(\frac{d}{\beta_2}\right)^{\frac{q+k+1}{2}}c^{k-q}}{q!k!(\alpha_1-q-1)!}$.

and,

$$J_2(q,k) = \frac{\Gamma(\alpha_1+1)\Gamma(\alpha_1+k-q)}{2\sqrt{\frac{d}{\beta_1\beta_2}}}\left(s+\frac{c}{\beta_1}\right)^{\frac{q-k-2\alpha_1}{2}} \times e^{\frac{d}{2\beta_2(c+\beta_1 s)}} W_{\frac{q-k-2\alpha_1}{2},\frac{q-k+1}{2}}\left(\frac{d}{\beta_2(c+\beta_1 s)}\right).$$

Proof. See the Appendix. ∎

For $c = 1$, the MGF in (7) reduces to

$$M_X(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_3(n,k,m)(b)^{\frac{n+m+1}{2}} \times \sum_{q=0}^{n+m+2}\binom{n+m+2}{q} \times \left(\frac{d}{b}\right)^q J_2(n,k,m,q) \tag{10}$$

where $c_3(n,k,m) = \frac{b^{k-m}\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{m!(k-m)!n!(\alpha_1-n-1)!}$.
and

$$J_2(n,k,m,q) = \frac{\sqrt{b\beta_1\beta_2}\,\Gamma(n+2)\Gamma(m+1)}{2d(-1)^{\alpha_1+k-q+1}} \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}}\Bigg\{e^{\frac{dp}{2b}} \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{d\left(p+\sqrt{p^2-\frac{4b}{\beta_1\beta_2}}\right)}{2b}\right) \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \times \left(\frac{d\left(p-\sqrt{p^2-\frac{4b}{\beta_1\beta_2}}\right)}{2b}\right)\Bigg\}\Bigg|_{p=s+\frac{1}{\beta_1}+\frac{b}{\beta_2}}.$$

which matches with the result derived in [18, Eq. (7)].

In the special case when $X_i \sim Exp(\gamma_i)$, $i = 1, 2$, the MGF in (7) reduces to

$$M_X(s) = 1 - 2s\sum_{q=0}^{2}\frac{\left(\frac{d}{bc}\right)^{q-1}(-1)^{q-2}}{(2-q)q!} \times \frac{d^{2-q}}{dp^{2-q}}\Bigg\{e^{\frac{dp}{2bc}} \times W_{-1,\frac{1}{2}}\left(\frac{d\left(p-\sqrt{p^2-4bc\gamma_1\gamma_2}\right)}{2bc}\right) \times W_{-1,\frac{1}{2}}\left(\frac{d\left(p+\sqrt{p^2-4bc\gamma_1\gamma_2}\right)}{2bc}\right)\Bigg\}\Bigg|_{p=s+c\gamma_1+b\gamma_2}. \tag{11}$$

Corollary 1 (*The Expected Value of $X = X_1X_2/(bX_1+cX_2+d)$*). *Let X_1 and X_2 be two independent Gamma RVs with parameters (α_1,β_1) and (α_2,β_2) respectively, [i.e., $X_i \sim Gamma(\alpha_i,\beta_i)$, $i = 1, 2$]. Let $b, d \geq 0$, and $c > 0$. Then, the expected value of the RV $X = X_1X_2/(bX_1+cX_2+d)$, $E(X)$, is given by*

Case ($b \neq 0, d \neq 0$):

$$E_X(x) = \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k}\sum_{q=0}^{n+m+2}\frac{(n+1)\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{(k-m)!(\alpha_1-n-1)!} \times \frac{b^{\frac{2k-m+n+2}{2}}c^{\frac{2\alpha_1-n+m}{2}}\left(\frac{d}{bc}\right)^q}{d(-1)^{\alpha_1+k-q+1}}\binom{n+m+2}{q} \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}}\Bigg\{e^{\frac{dp}{2bc}} \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{d\left(p-\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}}\right)}{2bc}\right) \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{d\left(p+\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}}\right)}{2bc}\right)\Bigg\}\Bigg|_{p=\frac{c}{\beta_1}+\frac{b}{\beta_2}}. \tag{12}$$

Proof. Given the MGF of X, $M_X(s)$, the expected value of X, $E(X)$, is expressed as

$$E(X) = \frac{d}{ds}M_X(s)_{|s=0}. \tag{13}$$

Using the MGF, $M_X(s)$, derived in (7) of Theorem 2 along with (11), $E(X)$ is given by (10). ∎

NB. For brevity, we have just gave here the case ($b \neq 0, d \neq 0$). For the two other cases (i.e., ($b \neq 0, d = 0$) and ($b = 0, d \neq 0$)), the MGF can be derived following the same line of thought.

In the special case when $X_i \sim Exp(\gamma_i)$, $i = 1, 2$, the expected value $E(X)$ in (12) reduces to

$$E_X(x) = 2\sqrt{\gamma_1\gamma_2}\sum_{q=0}^{2}\frac{\left(\frac{d}{bc}\right)^{q-1}}{(-1)^{2-q}(2-q)q!} \times \frac{d^{2-q}}{dp^{2-q}}\Bigg\{e^{\frac{dp}{2bc}} \times W_{-1,\frac{1}{2}}\left(\frac{d\left(p-\sqrt{p^2-4bc\gamma_1\gamma_2}\right)}{2bc}\right) \times W_{-1,\frac{1}{2}}\left(\frac{d\left(p+\sqrt{p^2-4bc\gamma_1\gamma_2}\right)}{2bc}\right)\Bigg\}\Bigg|_{p=c\gamma_1+b\gamma_2}. \tag{14}$$

Theorem 3 (*An Upper Bound for the Expected Value of $X = (1 + X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d))$*). *Let X_1 and X_2 be two independent Gamma RVs with parameters (α_1,β_1) and (α_2,β_2) respectively, [i.e., $X_i \sim Gamma(\alpha_i,\beta_i)$, $i = 1, 2$]. Let $b, d \geq 0$, and $c > 0$. Then, the expected value of the RV $X = (1 + X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d))$, $E(X)$, can be upper bounded as follows:*

$$E(X) \leq \log_2\left(\frac{(a+1)J_1+1}{aJ_1+1}\right), \tag{15}$$

where

Case $(b \neq 0, d \neq 0)$:

$$
\begin{aligned}
J_1 = & \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k}\sum_{q=0}^{n+m+2} \frac{(n+1)\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{(k-m)!n!(\alpha_1-n-1)!} \\
& \times \frac{b^{\frac{2k-m+n+2}{2}} c^{\frac{2\alpha_1-n+m}{2}} (\frac{d}{bc})^q}{d(-1)^{\alpha_1+k-q+1}} \binom{n+m+2}{q} \\
& \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}} \Bigg\{ e^{\frac{dp}{2bc}} W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \\
& \times \left(\frac{d(p-\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}})}{2bc}\right) \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \\
& \times \left(\frac{d(p+\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}})}{2bc}\right) \Bigg\}\Bigg|_{p=\frac{c}{\beta_1}+\frac{b}{\beta_2}} .
\end{aligned} \tag{16}
$$

Proof. See the Appendix. ■

NB. For brevity, we have just gave here the case $(b \neq 0, d \neq 0)$. For the two other cases (i.e., $(b \neq 0, d = 0)$ and $(b = 0, d \neq 0)$), the expected value of the RV $X = (1+X_1X_2/(aX_1X_2+bX_1+cX_2+d))$, $E(X)$, can be derived following the same line of thought.

In the special case when $X_i \sim Exp(\gamma_i), i = 1, 2, J_1$ in (16) reduces to

$$
\begin{aligned}
J_2 = & 2\sqrt{\gamma_1\gamma_2}\sum_{q=0}^{2} \frac{\left(\frac{d}{bc}\right)^{q-1}}{(2-q)q!} \times \frac{d^{2-q}}{dp^{2-q}} \Bigg\{ e^{\frac{dp}{2bc}} \\
& \times W_{-1,1}\left(\frac{d(p-\sqrt{p^2-4bc\gamma_1\gamma_2})}{2bc}\right) \\
& \times W_{-1,1}\left(\frac{d(p+\sqrt{p^2-4bc\gamma_1\gamma_2})}{2bc}\right) \Bigg\}\Bigg|_{p=c\gamma_1+b\gamma_2} .
\end{aligned} \tag{17}
$$

4. Application to the end-to-end performance evaluation of single-branch dual-hop AF relaying

Given the SNDR, Λ in (1), and using the key results presented in Section 3, we provide here exact closed-form expression for the outage probability and a tight upper bound for the ergodic capacity of the system in consideration.

4.1. The outage probability

The Outage probability denoted by $P_{out}(\lambda_{th})$ is defined as the probability that the channel fading makes the effective end-to-end SNDR fall below a certain threshold, λ_{th}, of the acceptable communication quality:

$$
P_{out}(\lambda_{th}) = Pr(\Lambda \leq \lambda_{th}) = F_\Lambda(\lambda_{th}). \tag{18}
$$

Using Theorem 1, the outage probability, $P_{out}(\lambda_{th})$, can be given by

$$
\begin{aligned}
P_{out}(\lambda_{th}) = & 1 - 2e^{-\left(\frac{b_2}{\beta_1}+\frac{b_1}{\beta_2}\right)\left(\frac{\lambda_{th}}{1-d\lambda_{th}}\right)} \\
& \times \sum_{j=0}^{\alpha_1-1}\sum_{n=0}^{\alpha_2-1}\sum_{k=0}^{j} c_4(j,n,k)\left(\frac{\lambda_{th}}{1-d\lambda_{th}}\right)^{\alpha_2+j} \\
& \times \left(b_1b_2+\frac{d(1-d\lambda_{th})}{\lambda_{th}}\right)^{\frac{n+k+1}{2}} \\
& \times K_{n-k+1}\left(2\sqrt{\frac{b_1b_2\lambda_{th}^2}{\beta_1\beta_2(1-d\lambda_{th})^2}+\frac{d\lambda_{th}}{\beta_1\beta_2(1-d\lambda_{th})}}\right)
\end{aligned} \tag{19}
$$

for $\lambda_{th} < \frac{1}{a}$ and $P_{out}(\lambda_{th}) = 1$ for $\lambda_{th} \geq \frac{1}{a}$, where $c_4(j,n,k) = \frac{b_1^{\alpha_2-n-1} b_2^{j-k} \beta_1^{\frac{k-n-1-2j}{2}} \beta_2^{\frac{n-k+1-2\alpha_2}{2}}}{k!(j-k)!n!(\alpha_2-n-1)!}$.

For $a = b = 1$ (i.e., CNA relay), the outage probability $P_{out}(\lambda_{th})$ in (19) reduces to [27, Eq. (27)]. The outage probability for the model proposed in [18] can be easily deduced from (19) by setting $k_1 = k_2 = 0, P_1 = P_2 = 1$, and $N_1 = N_2 = 1$. It is given by:

$$
\begin{aligned}
P_{out}(\lambda_{th}) = & 1 - 2e^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)\lambda_{th}} \sum_{j=0}^{\alpha_1-1}\sum_{n=0}^{\alpha_2-1}\sum_{k=0}^{j} c_5(j,n,k)\lambda_{th}^{\alpha_2+j} \\
& \times \left(a+\frac{b}{\lambda_{th}}\right)^{\frac{n+k+1}{2}} \\
& \times K_{n-k+1}\left(2\sqrt{\frac{(a\lambda_{th}+b)\lambda_{th}}{\beta_1\beta_2}}\right),
\end{aligned} \tag{20}
$$

where $c_5(j,n,k) = \frac{a^{\alpha_2-n-1}\beta_1^{\frac{k-n-1-2j}{2}}\beta_2^{\frac{n-k+1-2\alpha_2}{2}}}{k!(j-k)!n!(\alpha_2-n-1)!}$.

When both hops undergo Rayleigh fading, the outage probability $P_{out}(\lambda_{th})$ in (19) reduces to

$$
\begin{aligned}
P_{out}(\lambda_{th}) = & 1 - 2\sqrt{\gamma_1\gamma_2}e^{-(b_2\gamma_1+b_1\gamma_2)\left(\frac{\lambda_{th}}{1-d\lambda_{th}}\right)} \\
& \times \sqrt{\frac{b_1b_2\lambda_{th}^2}{(1-c\lambda_{th})^2}+\frac{c\lambda_{th}}{(1-c\lambda_{th})}} \\
& \times K_1\left(2\sqrt{\gamma_1\gamma_2\left(\frac{b_1b_2\lambda_{th}^2}{(1-d\lambda_{th})^2}+\frac{c\lambda_{th}}{(1-d\lambda_{th})}\right)}\right)
\end{aligned} \tag{21}
$$

for $\lambda_{th} < \frac{1}{d}$ and $P_{out}(\lambda_{th}) = 1$ for $\lambda_{th} \geq \frac{1}{d}$.

For $a = b = 1$ (i.e., CNA relay), the outage probability $P_{out}(\lambda_{th})$ in (21) reduces to [27, Eq. (29)].

For $k_1 = k_2 = 0, P_1 = P_2 = 1$, and $N_1 = N_2 = 1$, the outage probability $P_{out}(\lambda_{th})$ in (21) reduces to

$$
\begin{aligned}
P_{out}(\lambda_{th}) = & 1 - 2\sqrt{\gamma_1\gamma_2}e^{-(\gamma_1+a\gamma_2)\lambda_{th}} \frac{\sqrt{(a-b^2)\lambda_{th}^2+b\lambda_{th}}}{(1-b\lambda_{th})} \\
& \times K_1\left(2\sqrt{\gamma_1\gamma_2\lambda_{th}(a\lambda_{th}+b)}\right).
\end{aligned} \tag{22}
$$

For $a = 1$ and $b = 0$ (i.e., CA relay), the outage probability $P_{out}(\lambda_{th})$ in (22) reduces to

$$
P_{out}(\lambda_{th}) = 1 - 2\sqrt{\gamma_1\gamma_2}e^{-(\gamma_1+\gamma_2)\lambda_{th}}\lambda_{th} \times K_1\left(2\sqrt{\gamma_1\gamma_2}\lambda_{th}\right). \tag{23}
$$

For $a = 0$ and $b = 1$ (i.e., blind relay), the outage probability $P_{out}(\lambda_{th})$ in (22) reduces to

$$
P_{out}(\lambda_{th}) = 1 - 2\sqrt{\gamma_1\gamma_2}e^{-\gamma_1\lambda_{th}}\sqrt{\frac{\lambda_{th}}{(1-\lambda_{th})}} \times K_1\left(2\sqrt{\gamma_1\gamma_2\lambda_{th}}\right). \tag{24}
$$

4.2. The ergodic capacity

For a dual-hop AF relaying system, the ergodic capacity (in bits/channel use) can be expressed as

$$C_{erg} \triangleq \frac{1}{2} E\{log_2(1+\Lambda)\}, \tag{25}$$

where Λ is the end-to-end SNDR in (1).

Using Theorem 3, the ergodic capacity, C_{erg}, in (25) can be upper bounded as follows

$$C_{erg} \leq \frac{1}{2} log_2\left(\frac{(d+1)J_3+1}{dJ_3+1}\right), \tag{26}$$

where

$$J_3 = \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k}\sum_{q=0}^{n+m+2} \frac{(n+1)\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{n-m-1-2k}{2}}}{(k-m)!(\alpha_1-n-1)!} \times \frac{b_1^{\frac{2k-m+n+2}{2}} b_2^{\frac{2\alpha_1-n+m}{2}} (\frac{c}{b_1b_2})^q}{c(-1)^{\alpha_1+k-q+1}} \binom{n+m+2}{q} \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}} \Bigg\{ e^{\frac{cp}{2b_1b_2}} W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \times \left(\frac{c(p-\sqrt{p^2-\frac{4b_1b_2}{\beta_1\beta_2}})}{2b_1b_2}\right) \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \times \left(\frac{c(p+\sqrt{p^2-\frac{4b_1b_2}{\beta_1\beta_2}})}{2b_1b_2}\right)\Bigg\}\Bigg|_{p=\frac{b_2}{\beta_1}+\frac{b_1}{\beta_2}}, \tag{27}$$

for $(b \neq 0, d \neq 0)$.

NB. For brevity, we have just gave here the value of J_3 for the case $(b \neq 0, d \neq 0)$.

For $a = b = 1$ (i.e., CNA relay), J_3 in (27) reduces to [27, Eq. (34)].

An upper bound for the ergodic capacity for the model proposed in [18] can be easily deduced from (26) along with (27) by setting $k_1 = k_2 = 0$, $P_1 = P_2 = 1$, and $N_1 = N_2 = 1$. In this case J_3 in (27) becomes

$$J_4 = \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k}\sum_{q=0}^{n+m+2} \frac{(n+1)\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{(k-m)!(\alpha_1-n-1)!} \times a^{\frac{2k-m+n+2}{2}}\left(\frac{b}{a}\right)^q \times \binom{n+m+2}{q} \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}} \Bigg\{ e^{\frac{bp}{2a}} W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \left(\frac{b(p-\sqrt{p^2-\frac{4a}{\beta_1\beta_2}})}{2a}\right) \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}} \left(\frac{b(p+\sqrt{p^2-\frac{4a}{\beta_1\beta_2}})}{2a}\right)\Bigg\}\Bigg|_{p=\frac{1}{\beta_1}+\frac{a}{\beta_2}}. \tag{28}$$

When both hops undergo Rayleigh fading, J_3 in (27) reduces to

$$J_5 = 2\sqrt{\gamma_1\gamma_2}\sum_{q=0}^{2}\frac{\left(\frac{c}{b_1b_2}\right)^{q-1}(-1)^{q-2}}{(2-q)q!} \times \frac{d^{2-q}}{dp^{2-q}}\Bigg\{e^{\frac{cp}{2b_1b_2}}W_{-1,1}\left(\frac{c(p-\sqrt{p^2-4b_1b_2\gamma_1\gamma_2})}{2b_1b_2}\right) \times W_{-1,1}\left(\frac{c(p+\sqrt{p^2-4b_1b_2\gamma_1\gamma_2})}{2b_1b_2}\right)\Bigg\}\Bigg|_{p=b_2\gamma_1+b_1\gamma_2}. \tag{29}$$

For $k_1 = k_2 = 0$, $P_1 = P_2 = 1$, and $N_1 = N_2 = 1$, J_5 in (29) reduces to

$$J_6 = 2\sqrt{\gamma_1\gamma_2}\sum_{q=0}^{2}\frac{\left(\frac{b}{a}\right)^{q-1}}{(2-q)q!}\times\frac{d^{2-q}}{dp^{2-q}} \times \Bigg\{e^{\frac{bp}{2a}}W_{-1,1}\left(\frac{b(p-\sqrt{p^2-4a\gamma_1\gamma_2})}{2a}\right) \times W_{-1,1}\left(\frac{b(p+\sqrt{p^2-4a\gamma_1\gamma_2})}{2a}\right)\Bigg\}\Bigg|_{p=\gamma_1+a\gamma_2}. \tag{30}$$

4.3. Asymptotic SNR analysis

Now, we expatiate on the high-SNR regime, to get some insights and understandings on the fundamental impact of hardware impairments. The quantity $SNR_i = \frac{P_i\Lambda_i}{N_i}$ for $i = 1, 2$ is referred to as the ith hop average SNR. To facilitate the presentation, we assume that SNR_1, SNR_2 grow large with $SNR_1/SNR_2 = \mu$ for some ratio $\mu > 0$, such that the relaying gain G remains finite and strictly positive.

Corollary 2. *suppose SNR_1, SNR_2 grow large with $SNR_1 = \mu SNR_2$, for $0 < \mu < \infty$. The asymptotic SNDR satisfies*

$$\lim_{SNR_1,SNR_2\to\infty}\Lambda = \Lambda^* = \frac{1}{k_1^2+ak_2^2(1+k_1^2)} = \begin{cases}\frac{1}{k_1^2+k_2^2(1+k_1^2)}, & \text{for CNA \& CA relay configurations.}\\ \frac{1}{k_1^2}, & \text{for Blind relay configuration.}\end{cases} \tag{31}$$

Proof. Referring back to (1) and (2), and re-writing the SNDR, Λ in terms of SNR_1 and SNR_2, then taking the limit $SNR_1, SNR_2 \to \infty$, we can easily see that the SNDR converges to (31). ■

Corollary 3. *suppose SNR_1, SNR_2 grow large with $SNR_1 = \mu SNR_2$, for $0 < \mu < \infty$. The asymptotic OP with non-ideal hardware satisfies*

$$\lim_{SNR_1,SNR_2\to\infty} P_{out}(x) = P^* = \begin{cases}0, & x \leq \frac{1}{k_1^2+ak_2^2(1+k_1^2)}\\ 1, & x > \frac{1}{k_1^2+ak_2^2(1+k_1^2)}\end{cases} \tag{32}$$

Proof. The proof of the result in Corollary 3 follows a similar line of reasoning as in it of Corollary 2. ■

From Corollaries 2 and 3, we can deduct some conclusions. Firstly, from (32), we can clearly see that for a threshold x smaller than a SNDR ceiling value Λ^*, the outage probability $P_{out}(x)$ is equal to zero, while it is always equal to one for thresholds larger than this SNDR ceiling value Λ^*, which considerably limits the performance of the system in consideration. Secondly, the SNDR ceiling value Λ^* in (31) for $a = 1$ is a symmetric function of k_1 and k_1, and it is inversely proportional to their squares and it is independent of h_1, h_2 the fading distributions of the two hops. The SNDR ceiling value is around twice large for Blind relay configuration as for CNA/CA relay configurations[1]; which implies that the former can handle practical applications with twice as large hardware impairment constraints as the latter.

Corollary 4. *suppose SNR_1, SNR_2 grow large with $SNR_1 = \mu SNR_2$, for $0 < \mu < \infty$. The ergodic capacity satisfies*

$$\lim_{SNR_1, SNR_2 \to \infty} C_{erg} = C^*_{erg} = \frac{1}{2} log_2 \left(1 + \frac{1}{k_1^2 + a k_2^2 (1 + k_1^2)} \right) \tag{33}$$

Proof. Taking the $\lim_{SNR_1, SNR_2 \to \infty}$ of (25) and considering the dominated convergence theorem, we can move the limit inside the expectation operator to finally get the result in (33).

Eq. (33) indicates the presence of a capacity ceiling in the high-SNR regime.

4.4. Numerical results

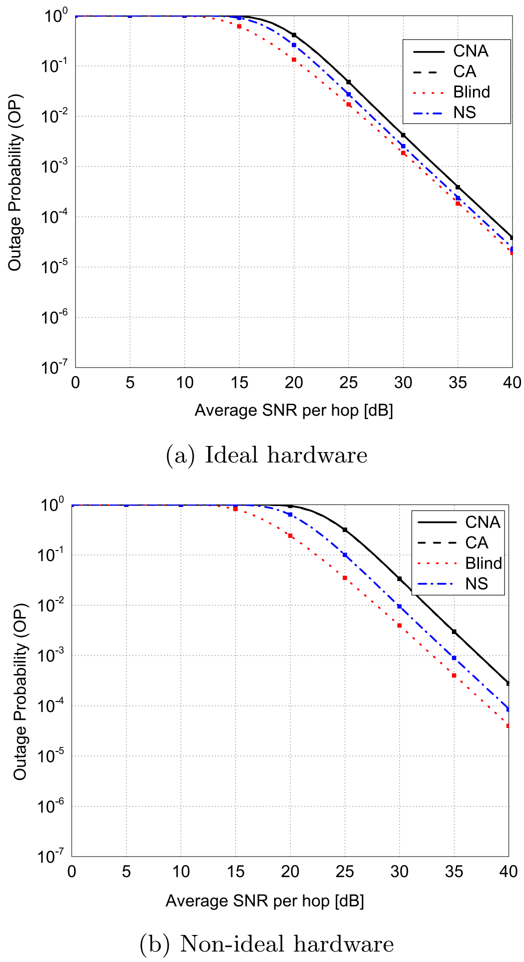
In this subsection, the theoretical results for the outage probability and the ergodic capacity are verified through Monte Carlo simulations which was carried out by averaging 10^7 sample point.

4.4.1. Different channel fading conditions

First, in Fig. 2, the outage probability, $P_{out}(\lambda_{th})$, as a function of the average SNR for a threshold $\lambda_{th} = 2^5 - 1 = 31$ (i.e., 5 bits/channel use) is shown. We consider both ideal hardware scenario and non-ideal hardware scenario with $k_1 = k_2 = 0.1$, independent Nakagami-m fading links (with $\alpha_1 = \alpha_2 = 2$, $\beta_1 = \beta_2 = 1$) with four relay configurations: CNA ($a = 1, b = 1$), CA ($a = 1, b = 0$), Blind ($a = 0, b = 1$), and NS ($a = 0.5, b = 1$). The two hops have the same average SNR (i.e., $SNR_1 = SNR_2$, where $SNR_i = (P_i \Lambda_i)/N_i$).

From this figure, we can clearly see that: (1) The analytical expressions (represented by the lines) match the results of Monte Carlo simulations (represented by markers). (2) There is a remarkable performance loss caused by hardware impairments. Precisely, CNA and CA relaying (which have almost the same performance) experience a loss of around 5 dB, NS loses around 3 dB while blind relaying loses just about 2 dB, which indicates that the latter is more robust to hardware impairments. (3) For the same SNR, CNA and CA relaying have the highest outage probability. Blind relaying suffers the least outage probability, while NS which is less conservative of transmit power compared to CNA and CA relaying, and more conservative compared to blind relaying has an outage probability in between.

In Fig. 3, we show the OP, $P_{out}(x)$ for a threshold $x = 3$ as a function of the shape parameters α_1, α_2 of the Nakagami-*m* fading distributions for the four different relay configurations (i.e., CNA, CA, Blind and NS). Three different asymmetric average SNR setups where $SNR_1 = \mu SNR_2$, for $\mu \in \{\frac{1}{5}, 1, 5\}$ are checked, while the largest of the SNRs is fixed to 20 dB. Ideal hardware and non-ideal hardware with $k_1 = k_2 = 0.1$ are considered. From this figure, we can observe that increasing the shape parameters will decrease OP and hence improving the system's performance. Besides, we can note that it is much better to have a symmetric SNR setup (i.e., $\mu = 1$) than an asymmetric one (i.e., $\mu \neq 1$). In an asymmetric setup, we can note from the simulation results that it is better to have a strong first hop and a weak second hop ($\mu = 5$) than the opposite (i.e., $\mu = \frac{1}{5}$) for blind and NS relay configurations. For CNA and CA relay configurations which have almost the same OP behavior, there is no difference in taking $\mu = 5$ or $\mu = \frac{1}{5}$. This can be explained by the symmetry in the expressions of the SNDR of the CNA and CA relay configurations.[2]



(a) Ideal hardware

(b) Non-ideal hardware

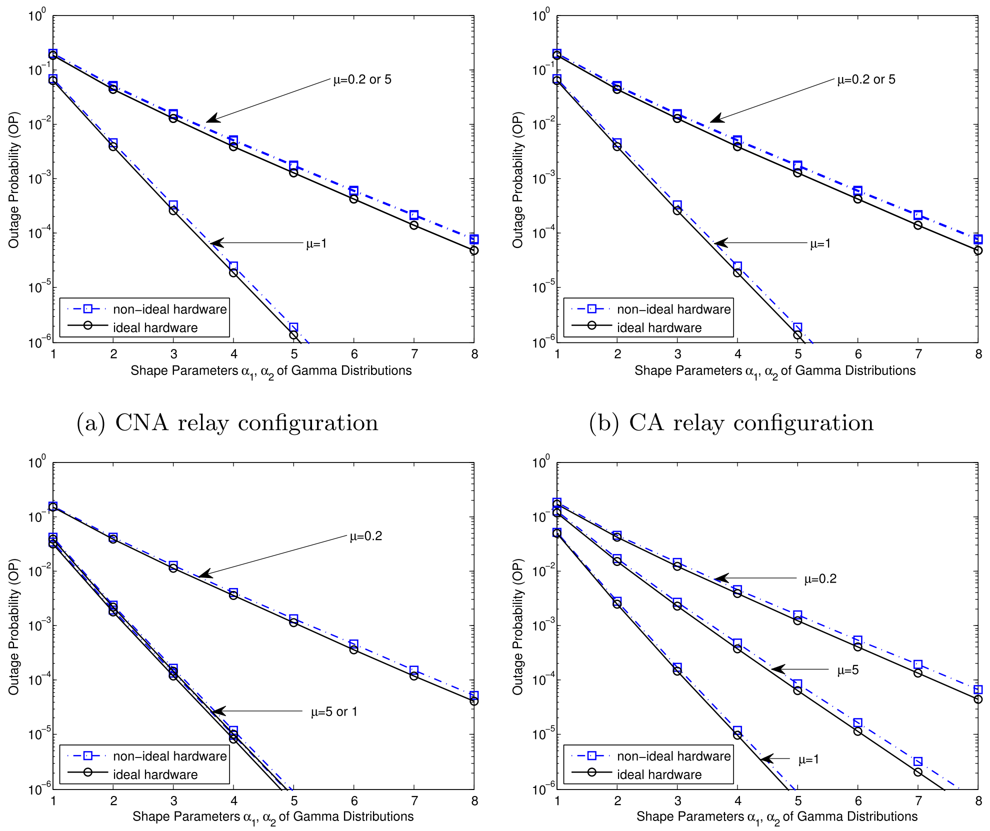
Fig. 2. Outage probability, $P_{out}(\lambda_{th})$, for AF relaying (ideal and non-ideal hardware cases). Simulated and analytical results for CNA, CA, Blind and NS relay configurations.

4.4.2. SNDR and capacity ceilings

To illustrate the existence of SNDR ceiling, we show in Fig. 4 the OP, $P_{out}(x)$, as a function of the threshold x (in dB) for both ideal and non-ideal hardware with impairments of level $k_1 = k_2 = 0.15$, for the four different relay configurations (CNA, CA, Blind and NS). We consider independent Nakagami-*m* fading channels with $\alpha_1 = \alpha_2 = 2$, and the same average SNR of 30 dB at both channels. For all relay configurations, the outage probability is slightly degraded by hardware impairments, at low threshold values. As the threshold x increases, the ideal hardware case smoothly converges toward 1, while the practical non-ideal case encounters a rapid convergence to its corresponding SNR ceiling for each relay configuration derived in Corollary 2. As can be seen from this figure, Blind relay configuration is more resilient to hardware impairments and its SNDR ceiling is approximately twice as large as that of CNA and CA

[1] Let us take $k_1 = k_1 = k > 0$, so $\Lambda^*_{CNA/CA} = \frac{1}{2k^2 + k^4} < \frac{1}{2k^2}$ and $\Lambda^*_{Blind} = \frac{1}{k^2}$, which gives $\Lambda^*_{Blind} > 2\Lambda^*_{CNA/CA}$.

[2] The SNDR for the CNA and CA relay configurations can be written as functions of SNR_1 and SNR_2 as follows: $\Lambda_{CNA} = \frac{1}{(k_1^2 + k_2^2(1+k_1^2)) + (1+k_1^2)\frac{1}{SNR_2} + (1+k_2^2)\frac{1}{SNR_1} + \frac{1}{SNR_1}\frac{1}{SNR_2}}$. $\Lambda_{CA} = \frac{1}{(k_1^2 + k_2^2(1+k_1^2)) + (1+k_1^2)\frac{1}{SNR_2} + \frac{1}{SNR_1} + \frac{1}{SNR_1}\frac{1}{SNR_2}}$.



(a) CNA relay configuration

(b) CA relay configuration

(c) Blind relay configuration

(d) NS relay configuration

Fig. 3. Outage probability, $P_{out}(3)$, for AF relaying (CNA, CA, Blind and NS relay configurations) with ideal and non-ideal hardware cases. $k_1^2 = k_2^2 = 1$. Different shape parameters α_1, α_2 are considered.

relay configurations. As expected, NS relay configuration has an in between performance. CNA and CA configurations have the same behavior.

Fig. 5 shows the ergodic capacity of CNA ($a = 1, b = 1$), Blind ($a = 0, b = 1$), and NS ($a = 0.5, b = 1$) AF relaying as a function of the average SNR. The two links undergo independent Nakagami-m fading with $\alpha_1 = \alpha_2 = 2$, $\beta_1 = \beta_2 = 1$. Ideal hardware scenario and non-ideal hardware scenario with $k_1 = k_2 \in \{0.05, 0.15\}$ are considered. It confirms the strong influence of hardware impairments at high SNR, where we can clearly see the saturation of the ergodic capacity. Blind relaying, which suffered the least outage probability, exhibits, as expected, the highest ergodic capacity, while CNA relaying has the lowest ergodic capacity. The curve for the NS case lies between those of blind and CNA relaying.

5. Conclusion

In this paper, a general E2E SNDR model for single-branch dual-hop AF relaying was presented. This model encompasses standard relay configurations, namely CNA, CA, and blind relaying as special cases. A unified mathematical framework was then developed, and its analytically-proved key results were used to derive an exact analytical expression for the outage probability and an upper bound for the ergodic capacity.

Appendix. Proofs of the results in Section 3

The proofs of Theorems 1–3 are presented here in a succinct form. Let X_1 and X_2 be two independent Gamma RVs with parameters (α_1, β_1) and (α_2, β_2), respectively, [i.e., $X_i \sim Gamma(\alpha_i, \beta_i)$, $i = 1, 2$]. Their CDF and PDF are given, for $i \in \{1, 2\}$, by

$$F_{X_i}(x_i) = 1 - e^{-\frac{x_i}{\beta_i}} \sum_{j=0}^{\alpha_i - 1} \frac{(x_i/\beta_i)^j}{j!} U(x_i);$$

$$f_{X_i}(x_i) = \frac{x_i^{\alpha_i - 1} e^{-\frac{x_i}{\beta_i}}}{\Gamma(\alpha_i)\beta_i^{\alpha_i}} U(x_i). \quad (34)$$

A.1. Proof of Theorem 1: CDF of $X = X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d)$

Let X_1 be a new RV given by

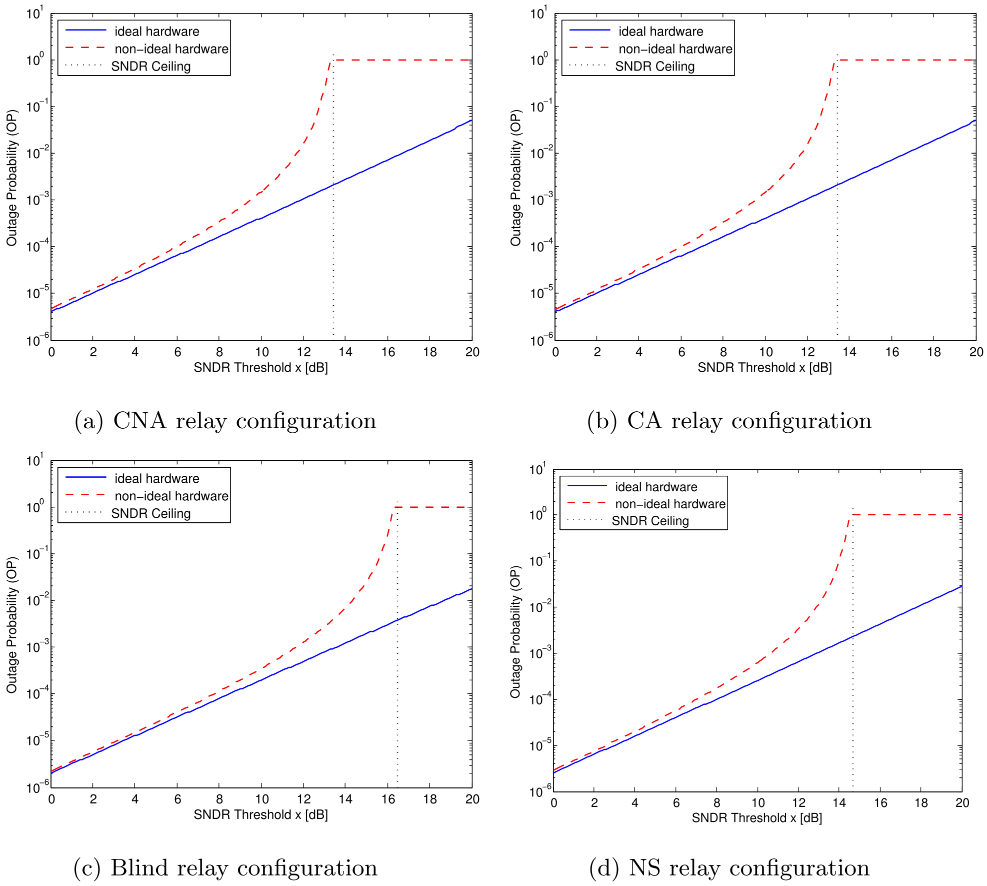
$$X = \frac{X_1X_2}{(aX_1X_2 + bX_1 + cX_2 + d)}. \quad (35)$$

Let $Pr(X)$ denotes the probability of the event X given in (35). The CDF of X, $F_X(x)$, is expressed as

$$F_X(x) = Pr(X \leq x). \quad (36)$$

The CDF of X, $F_X(x)$, in (36) depends on X which is a function of both X_1 and X_2. By applying the law of total probability to condition on X_2, we get

$$F_X(x) = 1 - \int_0^{+\infty} (1 - Pr(X \leq x/x_2)) f_{X_2}(x_2) dx_2$$
$$= 1 - \begin{cases} \int_{\frac{bx}{1-ax}}^{+\infty} \left(1 - F_{X_1}\left(\frac{(cx_2+d)x_2}{x_2(1-ax)-bx}\right)\right) f_{X_2}(x_2)dx, & x < \frac{1}{a} \\ \int_0^{+\infty} (1-1) f_{X_2}(x_2)dx = 0, & x \geq \frac{1}{a}. \end{cases} \quad (37)$$



(a) CNA relay configuration

(b) CA relay configuration

(c) Blind relay configuration

(d) NS relay configuration

Fig. 4. Outage probability, $P_{out}(\lambda_{th})$, for AF relaying (CNA, CA, Blind and NS relay configurations) with ideal and non-ideal hardware cases for different threshold values. As proved in Corollaries 2 and 3, there exist ceilings in the case of non-ideal hardware.

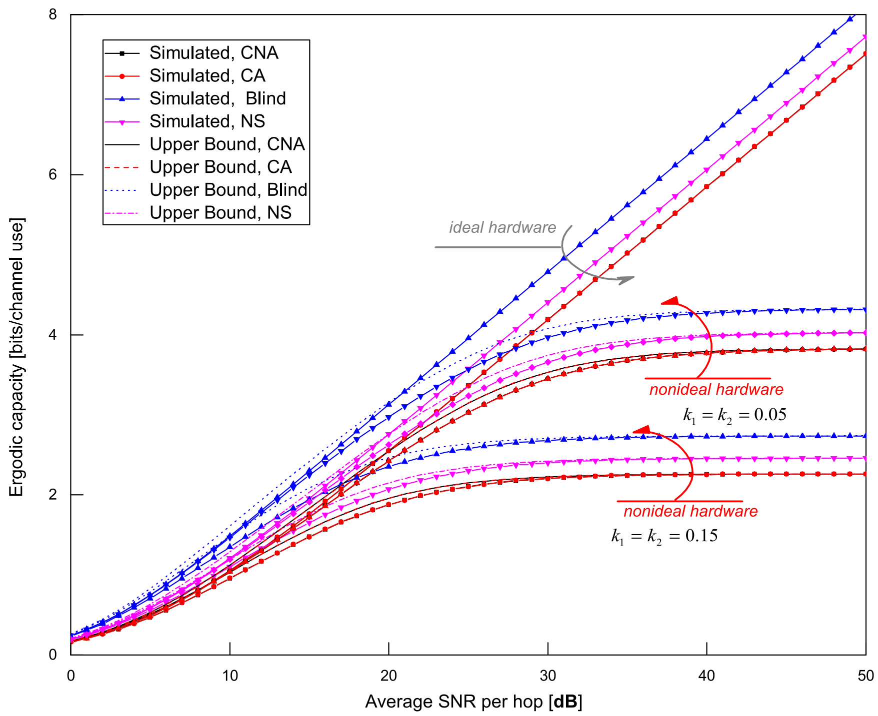


Fig. 5. Ergodic capacity, C_{erg} for AF relaying (ideal and non-ideal hardware cases). (CNA, blind and NS relay configurations).

For $x < \frac{1}{a}$, we have

$$F_X(x) = 1 - \int_{\frac{bx}{1-ax}}^{+\infty} \left(1 - F_{X_1}\left(\frac{(cx_2+d)x_2}{x_2(1-ax)-bx}\right)\right) f_{X_2}(x_2)dx. \quad (38)$$

Putting $z = x_2 - \frac{bx}{(1-ax)}$, gives

$$F_X(x) = 1 - \int_{\frac{bx}{1-ax}}^{+\infty} \left(1 - F_{X_1}\left(\frac{cx}{(1-ax)} + \frac{\frac{bcx^2}{(1-ax)}+bx}{z(1-ax)}\right)\right)$$

$$\times f_{X_2}(z + \frac{bx}{(1-ax)})dz. \quad (39)$$

By plugging the CDF, and the PDF, defined in (24) in (39), we get

$$F_X(x) = 1 - \sum_{j=0}^{\alpha_1-1} \frac{e^{-\left(\frac{cx}{\beta_1(1-ax)}+\frac{bx}{\beta_2(1-ax)}\right)}}{j!\beta_1^j\beta_2^{\alpha_2}\Gamma(\alpha_2)}$$

$$\times \int_0^{+\infty} \left(\frac{cx}{(1-ax)} + \frac{\frac{bcx^2}{(1-ax)}+bx}{z(1-ax)}\right)^j \quad (40)$$

$$\times \left(z + \frac{bx}{(1-ax)}\right)^{\alpha_2-1} e^{-\frac{1}{z}\left(\frac{bcx^2}{\beta_1(1-ax)^2}+\frac{dx}{\beta_1(1-ax)}\right)-\frac{z}{\beta_2}}dz.$$

By performing binomial expansion on the first two terms in the integral in (40), then using the integral identity in [32, Eq. (3.471.9)], (40) can be simplified to (5). ■

A.2. Proof of Theorem 2: MGF of $X = X_1X_2/(bX_1+cX_2+d)$

Let X_1 be a new RV given by

$$X = \frac{X_1X_2}{(bX_1+cX_2+d)}. \quad (41)$$

Given the complementary CDF of X, $\bar{F}_X(x)$, the MGF of X, $M_X(s)$, can be expressed via integration by parts as

$$M_X(s) = 1 - s\int_0^{+\infty} \bar{F}_X(x)e^{-sx}dx. \quad (42)$$

Case ($b \neq 0, d \neq 0$):
$\bar{F}_X(x)$ can be easily deduced from Theorem 1. By setting $a = 0$ in (5), we get

$$\bar{F}_X(x) = 2e^{-(\frac{c}{\beta_1}+\frac{b}{\beta_2})x} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)x^{\alpha_1+n}\left(bc+\frac{d}{x}\right)^{\frac{k+m+1}{2}}$$

$$\times K_{k-m+1}\left(2\sqrt{\frac{bcx^2+dx}{\beta_1\beta_2}}\right). \quad (43)$$

Substituting (43) in (42) yields

$$M_X(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)\int_0^{+\infty} e^{-(s+\frac{c}{\beta_1}+\frac{b}{\beta_2})x}$$

$$\times \left(bc+\frac{d}{x}\right)^{\frac{k+m+1}{2}} x^{\alpha_1+k}K_{k-m+1}\left(2\sqrt{\frac{bcx^2+dx}{\beta_1\beta_2}}\right). \quad (44)$$

Since the integral in (44) is not mathematically tractable, we re-arrange its terms by separating out a factor $(bc+(d/x))^{k+m+2}$ in the integrand, and we perform binomial expansion on it. This leads to

$$M_X(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)$$

$$\times \sum_{q=0}^{k+m+1}\binom{k+m+1}{q}\left(\frac{d}{bc}\right)^q$$

$$\times \int_0^{+\infty} x^{-q+\alpha_1+n}\left(bc+\frac{d}{x}\right) \times K_{k-m+1}\left(2\sqrt{\frac{bcx^2+dx}{\beta_1\beta_2}}\right)$$

$$\times e^{-(s+\frac{c}{\beta_1}+\frac{b}{\beta_2})x}dx. \quad (45)$$

The integral in (45) can be re-arranged into the following Laplace transformation

$$I = (bc)^{-(\frac{k+m+3}{2})}\left(\frac{k+m+1}{2}\right)$$

$$\times \int_0^{+\infty} x^{\frac{k+m+1+2(-q+\alpha_1+n)}{2}} x\left(\frac{k+m+1}{2}\right)^{-1}$$

$$\times \left(x+\frac{d}{bc}\right)^{-\left(\frac{k+m+3}{2}\right)} \quad (46)$$

$$\times K_{2\left(\frac{k-m+1}{2}\right)}\left(2\sqrt{\frac{bc}{\beta_1\beta_2}}\left(x^2+\frac{d}{bc}x\right)^{\frac{1}{2}} e^{-\left(s+\frac{c}{\beta_1}+\frac{b}{\beta_2}\right)x}\right)dx.$$

By using identities in [33, Eq. (4.1.6)] and [33, Eq. (4.17.20)], (46) can be simplified to the following form

$$I = (bc)^{-\left(\frac{k+m+3}{2}\right)}\left(\frac{k+m+3}{2}\right)(-1)^{\alpha_1+n-q+1}$$

$$\times \frac{d^{\alpha_1+n-q+1}}{dp^{\alpha_1+n-q+1}}\left\{\frac{\sqrt{\beta_1\beta_2bc}}{2d}\Gamma(k+1)\Gamma(m+1)e^{\frac{dp}{2bc}}\right.$$

$$\times W_{-\frac{k+m+2}{2},\frac{k+m+2}{2}}\left(\frac{d\left(p-\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}}\right)}{2bc}\right)$$

$$\left.\times W_{-\frac{k+m+2}{2},\frac{k+m+2}{2}}\left(\frac{d\left(p+\sqrt{p^2-\frac{4bc}{\beta_1\beta_2}}\right)}{2bc}\right)\right\}\Bigg|_{p=s+\frac{c}{\beta_1}+\frac{b}{\beta_2}}. \quad (47)$$

Substituting (47) in (45) gives (7). ■
Case ($b \neq 0, d = 0$):
By setting $d = 0$, (44) can be reduced to

$$M_X(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)(bc)^{\frac{k+m+1}{2}}$$

$$\times \int_0^{+\infty} e^{-(s+\frac{c}{\beta_1}+\frac{b}{\beta_2})x} \quad (48)$$

$$\times x^{\alpha_1+k}K_{k-m+1}\left(2\sqrt{\frac{bc}{\beta_1\beta_2}}x\right).$$

Applying [32, Eq. (6.621.3)] to evaluate the integral in (48), and simplifying the result one will get (8).

Case ($b = 0, d \neq 0$):

Setting $b = 0$ in (44) and applying [32, Eq. (6.631.3)] with $\alpha = s + (c/\beta_1)$, $\nu = q - k + 1$, $\mu = 2\alpha_1 + k - q$, and $\beta = 2\sqrt{d/\beta_1\beta_2}$, we get (9).

A.3. Proof of Theorem 3: (An upper bound for the expected value of $X = log_2(1 + X_1X_2/(aX_1X_2 + bX_1 + cX_2 + d))$

Let X be a new RV given by

$$X = log_2\left(1 + \frac{X_1X_2}{(aX_1X_2 + bX_1 + cX_2 + d)}\right). \tag{49}$$

X can be re-written as

$$X = log_2\left(\frac{(a+1)Y + 1}{aY + 1}\right), \tag{50}$$

where $Y = \frac{X_1X_2}{bX_1 + cX_2 + d}$.

Then, the expected value of X is given by

$$E(X) = E\left(log_2\left(\frac{(a+1)Y + 1}{aY + 1}\right)\right). \tag{51}$$

Notice that the function $log_2\left(\frac{(a+1)Y+1}{aY+1}\right)$ is twice-differentiable, and its second derivative is

$$-\frac{(2a^2Y + 2a(Y+1) + 1)}{log_e(2)(aY+1)^2(aY+Y+1)^2} < 0. \tag{52}$$

Jensen's inequality could be then applied to get

$$E(X) \leq log_2\left(\frac{(a+1)E(Y) + 1}{aE(Y) + 1}\right), \tag{53}$$

where $E(Y)$ can be obtained using Corollary 1. ■

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